BINOMIAL THEOREM

1. If $(1+x)^{15} = C_0 + C_1x + C_2x^2 + \ldots + C_{15}x^{15}$, then $C_2 + 2C_3 + 3C_4 + \ldots + 14C_{15}$ is equal to
   a) 14.2^{14} b) $13.2^{14}+1$ c) $13.2^{14}-1$ d) None of these
2. If the coefficients of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in A.P., then
   a) $2n^2+9n+7=0$ b) $2n^2-9n+7=0$ c) $2n^2-9n-7=0$ d) None of these
3. If $|x| < \frac{1}{2}$, then the coefficient of x^r in the expansion of $\frac{1+2x}{(1-2x)^2}$, is
   a) $r2^r$ b) $(2r-1)2^r$ c) $r2^{2r+1}$ d) $(2r+1)2^r$
4. $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \ldots \binom{30}{20}\binom{30}{30}$ is equal to
   a) ${}^{30}C_{11}$ b) ${}^{60}C_{10}$ c) ${}^{30}C_{10}$ d) ${}^{65}C_{55}$
5. If $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + \cdots + a_{2n}x^{2n}$, then $a_0 + a_2 + a_4 + \cdots + a_{2n}$ is equal to
   a) $\frac{3^n+1}{2}$ b) $\frac{3^n-1}{2}$ c) $\frac{3^{n-1}+1}{2}$ d) $\frac{3^{n-1}-1}{2}$
6. If $C_0, C_1, C_2, \ldots\ldots C_n$ denote the binomial coefficient in the expansion of $(1+x)^n$, then
   $C_0 \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1}$ is equal to
   a) $\frac{2^{n+1}-1}{n+1}$ b) $\frac{2^n-1}{n}$ c) $\frac{2^{n-1}-1}{n-1}$ d) $\frac{2^{n+1}-1}{n+2}$
7. If the ratio of the 7th term from the beginning to the seventh term from the end in the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^x$ is $\frac{1}{6}$ then x, is
   a) 9 b) 6, 15 c) 12, 9 d) None of these
8. If in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in N$, sum of the coefficients of x^5 and x^{10} is zero, then $n =$
   a) 5 b) 10 c) 15 d) 20
9. The range of the values of the term independent of x in the expansion of $\left(x\sin^{-1}\alpha + \frac{\cos^{-1}\alpha}{x}\right)^{10}$, $\alpha \in [-1,1]$ is
   a) $\left[\frac{{}^{10}C_5 \cdot \pi^{10}}{2^5}, -\frac{{}^{10}C_5\pi^{10}}{2^{20}}\right]$ b) $\left[-\frac{{}^{10}C_5 \cdot \pi^{10}}{2^5}, \frac{{}^{10}C_5 \cdot \pi^{10}}{2^{20}}\right]$
   c) $\left[\frac{{}^{10}C_5 \cdot \pi^5}{2^5}, \frac{{}^{10}C_5 \cdot \pi^5}{2^{20}}\right]$ d) $\left[-\frac{{}^{10}C_5 \cdot \pi^5}{2^5}, \frac{{}^{10}C_5 \cdot \pi^5}{2^{20}}\right]$
10. $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \ldots + \binom{30}{20}\binom{30}{30}$ is equal to
    a) ${}^{30}C_{11}$ b) ${}^{60}C_{10}$ c) ${}^{30}C_{10}$ d) ${}^{65}C_{55}$
11. The rth terms in the expansion of $(a+2n)^n$ is
    a) $\frac{n(n+1)\ldots(n-r+1)}{r!}a^{n-r+1}(2x)^r$ b) $\frac{n(n-1)\ldots(n-r+2)}{(r-1)!}a^{n-r+1}(2x)^{r-1}$
    c) $\frac{n(n+1)\ldots(n-r)}{r+1}a^{n-r}(x)^r$ d) None of the above
12. The coefficient of t^{24} in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is
    a) ${}^{12}C_6 + 2$ b) ${}^{12}C_5$ c) ${}^{12}C_6$ d) ${}^{12}C_7$

13. The coefficient of x^n in the expansion of $\frac{(1+x)^2}{(1-x)^3}$, is

a) n^2+2n+1 b) $2n^2+n+1$ c) $2n^2+2n+1$ d) n^2+2n+2

14. The sum of the coefficient in the expansion of $(1+x-3x^2)^{3148}$ is

a) 8 b) 7 c) 1 d) -1

15. If C_r stands for ${}^nC_r$, then the sum of first $(n+1)$ terms of the series $a\,C_0-(a+d)C_1+(a+2d)C_2-(a+3d)C_3+\cdots$, is

a) $\frac{a}{2^n}$ b) $n\,a$ c) 0 d) None of these

16. The value of $\frac{18^3+7^3+3\cdot18\cdot7\cdot25}{3^6+6\cdot243\cdot2+15\cdot181\cdot4+20\cdot27\cdot8+15\cdot9\cdot16+6\cdot3\cdot32+64}$, is

a) 10 b) 1 c) 2 d) 20

17. The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$, is

a) 30 b) 60 c) 40 d) None of these

18. If $n=5$, then
$({}^nC_0)^2+({}^nC_1)^2+({}^nC_2)^2+\ldots+({}^nC_5)^2$ is equal to

a) 250 b) 254 c) 245 d) 252

19. The coefficient of x^{50} in the expression $(1+x)^{1000}+2\,x(1+x)^{999}+3\,x^2(1+x)^{998}+\cdots+1001\,x^{1000}$ is

a) ${}^{1000}C_{50}$ b) ${}^{1001}C_{50}$ c) ${}^{1002}C_{50}$ d) ${}^{1000}C_{51}$

20. For $|x|<1$, the constant term in the expansion of
$\frac{1}{(x-1)^2(x-2)}$ is

a) 2 b) 1 c) 0 d) $-\frac{1}{2}$

21. $1+\frac{2\cdot1}{3\cdot2}+\frac{2\cdot5}{3\cdot6}\left(\frac{1}{2}\right)^2+\frac{2\cdot5\cdot8}{3\cdot6\cdot9}\cdot\left(\frac{1}{2}\right)^3+\ldots$ is equal to

a) $2^{1/3}$ b) $3^{1/4}$ c) $4^{1/3}$ d) $3^{1/3}$

22. If in the expansion of $\left(3x-\frac{2}{x^2}\right)^{15}$ rth term is independent of x, then value of r is

a) 6 b) 10 c) 9 d) 12

23. If $(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n$, then the value of $\sum_{0\le r<s\le n}\sum(r+s)(C_r+C_s)$ is

a) $n^2.2^n$ b) $n.2^n$ c) $n^2.2^{2n}$ d) None of these

24. If $C_0, C_1, C_2, \ldots, C_n$ denote the binomial coefficient in the expansion of $(1+x)^n$, then the value of $a\,C_0+(a+b)C_1+(a+2\,b)C_2+\cdots+(a+n\,b)C_n$, is

a) $(a+nb)2^n$ b) $(a+nb)2^{n-1}$ c) $(2\,a+nb)2^{n-1}$ d) $(2\,a+nb)2^n$

25. $C_0C_r+C_1C_{r+1}+C_2C_{r+2}+\ldots+C_{n-r}C_n$ is equal to

a) $\frac{(2n)!}{(n-r)!\,(n+r)!}$

b) $\frac{n!}{r!\,(n+r)!}$

c) $\frac{n!}{(n-r)!}$

d) None of these

26. If the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a, is

a) $-\frac{7}{9}$ b) $-\frac{9}{7}$ c) $\frac{7}{9}$ d) $\frac{9}{7}$

27. The total number of terms in the expansion of $(x+a)^{100}+(x-a)^{100}$after simplification will be

a) 202 b) 51 c) 50 d) None of these

28. Coefficient of x^{19} in the polynomial $(x-1)(x-2)\ldots..(x-20)$is equal to

a) 210 b) -210 c) 20! d) None of these

29. The sum of the last eight coefficient in the expansion of $(1+x)^{15}$ is

a) 2^{16} b) 2^{15} c) 2^{14} d) None of these

30. The number of terms in the expansion of $(a+b+c)^n$ will be
a) $n+1$
b) $n+3$
c) $\dfrac{(n+1)(n+2)}{2}$
d) None of these

31. The coefficient of y in the expansion of $(y^2+c/y)^5$, is
a) $29\ c$ b) $10\ c$ c) $10\ c^3$ d) $20\ c^2$

32. The value of $(0.99)^{15}$ is
a) 0.8432 b) 0.8601 c) 0.8502 d) None of these

33. The sum of the coefficients in the expansion of $(x+y)^n$ is 4096. The greatest coefficient in the expansion is
a) 1024 b) 924 c) 824 d) 724

34. If in the expansion of $(1+x)^n$, the coefficient of rth and $(r+2)$th term be equal, then r is equal to
a) $2n$ b) $\dfrac{2n+1}{2}$ c) $\dfrac{n}{2}$ d) $\dfrac{2n-1}{2}$

35. If the second, third and fourth term in the expansion of $(x+a)^n$ are 240,720 and 1080 respectively, then the value of n is
a) 15 b) 20 c) 10 d) 5

36. The value of $\frac{1}{81^n}-\frac{10}{81^n}\,{}^{2n}C_1\,\frac{10^2}{81^n}\,{}^{2n}C_2-\frac{10^3}{81^n}2\,{}^{2n}C_3+\ldots+\frac{10^{2n}}{81^n}$ is
a) 2 b) 0 c) $\dfrac{1}{2}$ d) 1

37. If $(1+x+x^2)^n=\sum_{r=0}^{2n} a_r x^r$
then, $a_1-2a_2+3a_3\ldots-2na_{2n}$ is equal to
a) n b) $-n$ c) 0 d) $2n$

38. The coefficient of the middle term in the expansion of $(1+x)^{2n}$, is
a) $\dfrac{1\cdot3\cdot5\ldots(2n-1)}{n\,!}2^n$ b) $\dfrac{1\cdot3\cdot5\ldots(2n-1)}{(n\,!)^2}2^n$ c) $\dfrac{(2\,n)\,!}{(n\,!)^2}2^{2n}$ d) None of these

39. The constant term in the expansion of $(1+x)^{10}\left(1+\frac{1}{x}\right)^{12}$ is
a) ${}^{22}C_{10}$ b) 0 c) ${}^{22}C_{11}$ d) None of these

40. If $a_1=1$ and $a_n=na_{n-1}$ for all positive integer $n\geq2$, then a_5 is equal to
a) 125 b) 120 c) 100 d) 24

41. If $(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n$, then the value of $C_0+2C_1+3C_2+\ldots+(n+1)C_n$ will be
a) $(n+2)2^{n-1}$
b) $(n+1)2^n$
c) $(n+1)2^{n-1}$
d) $(n+2)2^n$

42. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n\in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then $n=$
a) 25 b) 20 c) 15 d) None of these

43. In the expansion of $(1+x+x^2+x^3)^6$, then coefficient of x^{14} is
a) 130 b) 120 c) 128 d) 125

44. The 14th term from the end in the expansion of $(\sqrt{x}-\sqrt{y})^{17}$ is
a) ${}^{17}C_5x^6(-\sqrt{y})^5$ b) ${}^{17}C_6(\sqrt{x})^{11}y^3$ c) ${}^{17}C_4x^{13/2}y^2$ d) None of these

45. The sum of the coefficients in the expansion of $(1+2x+3x^2+\ldots+nx^n)^2$ is
a) $\sum 1$ b) $\sum n$ c) $\sum n^2$ d) $\sum n^3$

46. If a_k is the coefficient of x^k in the expansion of $(1+x+x^2)^n$ for $k=0,1,2,\ldots,2n$ then
a) $-a_0$ b) 3^n c) $n\cdot3^{n+1}$ d) $n\cdot3^n$

47. The coefficient of x^n in the polynomial $(x+{}^nC_0)(x+3\,{}^nC_1)(x+5\,{}^nC_2)\ldots[x+(2n+1)\,{}^nC_n]$

a) $n.2^n$ b) $n.2^{n+1}$ c) $(n+1)2^n$ d) $n.2^n+1$

48. ${}^{n-2}C_r+2\,{}^{n-2}C_{r-1}+{}^{n-2}C_{r-2}$ equals

a) ${}^{n+1}C_r$ b) ${}^nC_r$ c) ${}^nC_{r+1}$ d) ${}^{n-1}C_r$

49. For $|x|<1$, the constant term in the expansion of $\frac{1}{(x-1)^2(x-2)}$ is

a) 2 b) 1 c) 0 d) $-\frac{1}{2}$

50. Coefficient of x in the expansion of $\left(x^2+\frac{a}{x}\right)^5$ is

a) $9a^2$ b) $10a^3$ c) $10a^2$ d) $10a$

51. $\frac{1}{n!}+\frac{1}{2!\,(n-2)\,!}+\frac{1}{4!\,(n-4)\,!}+\cdots$ is equal to

a) $\frac{2^{n-1}}{n\,!}$ b) $\frac{2^n}{(n+1)\,!}$ c) $\frac{2^n}{n!}$ d) $\frac{2^{n-2}}{(n-1)\,!}$

52. The greatest coefficient in the expansion of $(1+x)^{10}$, is

a) $\frac{10\,!}{5\,!\,6\,!}$ b) $\frac{10\,!}{(5\,!)^2}$ c) $\frac{10\,!}{5\,!\,7!}$ d) None of these

53. In the expansion of $\left(\frac{a}{x}+bx\right)^{12}$, the coefficient of x^{-10} will be

a) $12a^{11}$ b) $12b^{11}a$ c) $12a^{11}b$ d) $12a^{11}b^{11}$

54. The coefficient of x^{10} in the expansion of $(1+x^2-x^3)^8$, is

a) 476 b) 496 c) 506 d) 528

55. If the $(r+1)^{\text{th}}$ term in the expansion of $\left\{\sqrt[3]{\frac{a}{\sqrt{b}}}+\sqrt{\frac{b}{\sqrt[3]{a}}}\right\}^{21}$ contains a and b to one and the same power, then the value of r, is

a) 9 b) 10 c) 8 d) 6

56. The $(r+1)$th term in the expansion of $(1-x)^{-4}$ will be

a) $\frac{x^r}{r!}$ b) $\frac{(r+1)(r+2)(r+3)}{6}x^r$

c) $\frac{(r+2)(r+3)}{2}x^r$ d) None of these

57. If $y=\frac{1}{3}+\frac{1\cdot3}{3\cdot6}+\frac{1\cdot3\cdot5}{3\cdot6\cdot9}+\cdots$, then the value of $y^2+2\,y$ is

a) 2 b) -2 c) 0 d) None of these

58. Let $S(k)=1+3+5+\ldots+(2k-1)=3+k^2$. Then, which of the following is true?

a) $S(1)$ is correct

b) $S(k)\Rightarrow S(k+1)$

c) $S(k)\nRightarrow S(k+1)$

d) Principle of mathematical induction can be used to prove the formula

59. The number of irrational terms in the expansion of $(5^{1/6}+2^{1/8})^{100}$ is

a) 96 b) 97 c) 98 d) 99

60. If the rth term in the expansion of $(x/3-2/x^2)^{10}$ contains x^4, then r is equal to

a) 2 b) 3 c) 4 d) 5

61. When $32^{(32)^{(32)}}$ is divided by 7, then the remainder is

a) 2

b) 8

c) 4

d) None of these

62. The value of x, for which the 6th term in the expansion of $\left\{2^{\log_2\sqrt{(9^{x-1}+7)}}+\frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right\}^7$ is 84, is equal to

a) 4 b) 3 c) 2 d) 5

63. If $P(n): 2+4+6+\cdots+(2n), n \in N$, then
$P(k)=k(k+1)+2$implies
$P(k+1)=(k+1)(k+2)+2$
is true for all $k \in N$. So, statement $P(n)=n(n+1)+2$ is true for
a) $n \geq 1$ b) $n \geq 2$ c) $n \geq 3$ d) None of these

64. The number of terms in the expansion of $(1+2x+x^2)^{20}$, when expanded in descending powers of x, is
a) 20 b) 21 c) 40 d) 41

65. The binomial coefficients which are in decreasing order are
a) ${}^{15}C_5, {}^{15}C_6, {}^{15}C_7$ b) ${}^{15}C_{10}, {}^{15}C_9, {}^{15}C_8$ c) ${}^{15}C_6, {}^{15}C_7, {}^{15}C_8$ d) ${}^{15}C_7, {}^{15}C_6, {}^{15}C_5$

66. $10^n+3(4^{n+2})+5$ is divisible by $(n \in N)$
a) 7 b) 5 c) 9 d) 17

67. $\frac{{}^8C_0}{6}= {}^8C_1+{}^8C_2\cdot 6-{}^8C_3\cdot 6^2+\cdots+{}^8C_8\cdot 6^7$ is equal to
a) 0 b) 6^7 c) 6^8 d) $\frac{5^8}{6}$

68. If the coefficients of the second, third and fourth terms in the expansion of $(1+x)^n$ are in AP, then n is equal to
a) 7
b) 2
c) 6
d) None of these

69. The expansion of $(8-3x)^{3/2}$ in terms of powers of x is valid only if
a) $x>\frac{8}{3}$ b) $|x|<\frac{8}{3}$ c) $x<\frac{3}{8}$ d) $x<\frac{8}{3}$

70. If ${}^nC_0, {}^nC_1, {}^nC_2 \ldots. {}^nC_n$ denote the coefficient of the binomial expansion $(1+x)^n$, then the value of $C_1+3C_3+5C_5+\ldots$is
a) $n2^{n-2}$ b) $n2^{n-1}$ c) $(n+1)2^n$ d) $(n+2)2^{n-1}$

71. The value of x in the expansion $[x+x^{\log_{10} x}]^5$, if the third term in the expansion is 1000000, is
a) 10 b) 11 c) 12 d) None of these

72. ${}^nC_0-\frac{1}{2}{}^nC_1+\frac{1}{3}{}^nC_2-\ldots+(-1)^n\frac{{}^nC_n}{n+1}$ is equal to
a) n b) $\frac{1}{n}$ c) $\frac{1}{n+1}$ d) $\frac{1}{n-1}$

73. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9, is
a) 0 b) 2 c) 7 d) 8

74. The number of rational terms in the expansion of $\left(\sqrt[3]{4}+\frac{1}{\sqrt[4]{6}}\right)^{20}$, is
a) 3 b) 18 c) 4 d) 16

75. The number of terms in the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n, n \in N$, is
a) $2n$ b) $3n$ c) $2n+1$ d) $3n+1$

76. The digit at the unit place in the number $19^{2005}+11^{2005}-9^{2005}$ is
a) 2 b) 1 c) 0 d) 8

77. The coefficient of the middle term in the expansion of $(x+2y)^6$ is
a) ${}^6C_3$ b) $8({}^6C_3)$ c) $8({}^6C_5)$ d) ${}^6C_4$

78. The coefficient of x^{-17} in the expansion of
$\left(x^4-\frac{1}{x^3}\right)^{15}$ is
a) ${}^{15}C_{11}$ b) ${}^{15}C_{12}$ c) $-{}^{15}C_{11}$ d) $-{}^{15}C_3$

79. If $\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{\sqrt{4-x}}$ is approximately equal to $a+bx$ for small values of x, then (a, b) is equal to

a) $\left(1, \frac{35}{24}\right)$ b) $\left(1, -\frac{35}{24}\right)$ c) $\left(2, \frac{35}{12}\right)$ d) $\left(2, -\frac{35}{12}\right)$

80. If ${}^{18}C_{15} + 2({}^{18}C_{16}) + {}^{17}C_{16} + 1 = {}^{n}C_3$, then n is equal to
a) 19 b) 20 c) 18 d) 24

81. $49^n + 16n - 1$ is divisible by
a) 3 b) 19 c) 64 d) 29

82. In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is
a) 0 b) 2^{49} c) 2^{50} d) 2^{51}

83. The number of terms in the expansion of $(x+y+z)^{10}$, is
a) 11 b) 33 c) 66 d) 1000

84. If $\alpha = \frac{5}{2!\,3} + \frac{5 \cdot 7}{3!\,3^2} + \frac{5 \cdot 7 \cdot 9}{4!\,3^2} + \ldots\ldots$, then $\alpha^2 + 4\alpha$ equal to
a) 21 b) 23 c) 25 d) 27

85. If $|x| < \frac{1}{2}$, then the coefficient of x^r in the expansion of $\frac{1+2x}{(1-2x)^2}$, is
a) $r2^r$ b) $(2r-1)2^r$ c) $r2^{2r+1}$ d) $(2r+1)2^r$

86. The coefficient of $x^n y^n$ in the expansion of
$\{(1+x)(1+y)(x+y)\}^n$, is
a) $\sum_{r=0}^{n} C_r^2$ b) $\sum_{r=0}^{n} C_{r+2}^2$ c) $\sum_{r=0}^{n} C_{r+3}^2$ d) $\sum_{r=0}^{n} C_r^3$

87. If $(1+x+x^2)^n = C_0 + C_1x + C_2x^2 + \cdots$, then the value of $C_0C_1 - C_1C_2 + C_2C_3 - \cdots$, is
a) 3^n b) $(-1)^n$ c) 2^n d) None of these

88. If a, b, c are in AP, then the sum of the coefficients of $\{1 + (ax^2 - 2bx + c)^2\}^{1973}$ is
a) -2 b) -1 c) 0 d) 1

89. If the second term in the expansion $\left[\sqrt[13]{a} + \frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\frac{{}^nC_3}{{}^nC_2}$ is
a) 4 b) 3 c) 12 d) 6

90. If $n > 1$, then $(1+x)^n - nx - 1$ is divisible by
a) $2x$ b) x^2 c) x^3 d) x^4

91. The coefficient of $x^6 a^{-2}$ in the expansion of $\left(\frac{x^2}{a} - \frac{a}{x}\right)^{12}$, is
a) ${}^{12}C_6$ b) $-{}^{12}C_5$ c) 0 d) None of these

92. If $(5 + 2\sqrt{6})^n = I + f$; $n, I \in N$ and $0 \le f < 1$, then I equals
a) $\frac{1}{f} - f$ b) $\frac{1}{1+f} - f$ c) $\frac{1}{1+f} + f$ d) $\frac{1}{1-f} - f$

93. If $n \in N, n > 1$, then value of $E = a - {}^nC_1(a-1) + {}^nC_2(a-2) + \ldots + (-1)^n(a-n)\,{}^nC_n$ is
a) a
b) 0
c) a^2
d) 2^n

94. If a_r is the coefficient of x^{r-1} in $(1+x)^n + (1+x)^{n+1} + \ldots + (1+x)^{n+k}$ $(n < r - 1 \le n + k)$, then $\sum_{r=0}^{n+k+1}(-1)^r a_r$ is equal to
a) 0
b) $n+k+1$
c) $(n+k+1)!$
d) ${}^{n+k+1}C_r$

95. The sum of $1 + n\left(1 - \frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1 - \frac{1}{x}\right)^2 + \cdots \infty$, will be
a) x^n b) x^{-n} c) $\left(1 - \frac{1}{x}\right)^n$ d) None of these

96. If $T_0, T_1, T_2, \dots, T_n$ represents the terms in the expansion of $(x+a)^n$, then $(T_0 - T_2 + T_4 - \dots)^2 + (T_1 - T_3 + T_5 - \dots)^2$ is equal to

a) $(x^2 + a^2)$
b) $(x^2 + a^2)^n$
c) $(x^2 + a^2)^{1/n}$
d) $(x^2 + a^2)^{-1/n}$

97. If the coefficient of $(2r+1)$th term and $(r+2)$th term in the expansion of $(1+x)^{43}$ are equal, then r is equal to

a) 12
b) 14
c) 16
d) 18

98. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots C_n x^n$, then $C_0^2 + C_1^2 + C_2^2 + C_3^2 + \dots + C_n^2$ is equal to

a) $\frac{n!}{n!\,n!}$
b) $\frac{(2n)!}{n!\,n!}$
c) $\frac{(2n)!}{n!}$
d) None of these

99. If n is a positive integer and $C_k = {}^nC_k$, then $\sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2$ equals

a) $\frac{n(n+1)(n+2)}{12}$
b) $\frac{n(n+1)^2(n+2)}{12}$
c) $\frac{n(n+1)(n+2)^2}{12}$
d) None of these

100. The value of
$1 \times 2 \times 3 \times 4 + 2 \times 3 \times 4 \times 5 + 3 \times 4 \times 5 \times 6 + \cdots$
$+ n(n+1)(n+2)(n+3)$, is

a) $\frac{1}{5}(n+1)(n+2)(n+3)(n+4)(n+5)$
b) $\frac{1}{5}n(n+1)(n+2)(n+3)(n+4)$
c) $\frac{1}{5}n(n+1)(n+2)(n+3)(n+4)$
d) ${}^{n+4}C_5$

101. The coefficient of $x^8 y^6 z^4$ in the expansion of $(x+y+z)^{18}$, is not equal to

a) ${}^{18}C_{14} \times {}^{14}C_8$
b) ${}^{18}C_{10} \times {}^{10}C_6$
c) ${}^{18}C_6 \times {}^{12}C_8$
d) ${}^{18}C_6 \times {}^{14}C_6$

102. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$, is

a) 900
b) 909
c) 990
d) 999

103. If the sum of the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ is a and if the sum of the coefficients in the expansion of $(1+x^2)^n$ is b, then

a) $a = 3b$
b) $a = b^3$
c) $b = a^3$
d) None of these

104. For $n \in N$, $10^{n-2} \geq 81n$ is

a) $n > 5$
b) $n \geq 5$
c) $n < 5$
d) $n > 8$

105. The first 3 terms in the expansion of $(1+ax)^n$ $(n \neq 0)$ are $1, 6x$, and $16x^2$. Then, the value of a and n are respectively

a) 2 and 9
b) 3 and 2
c) $\frac{2}{3}$ and 9
d) $\frac{3}{2}$ and 6

106. If the binomial expansion of $(a + bx)^{-2}$ is $\frac{1}{4} - 3x + \cdots$, then $(a, b) =$

a) $(2, 12)$
b) $(2, 8)$
c) $(-2, -12)$
d) None of these

107. In the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$, the coefficient of x^{39}, is

a) 1365
b) -1365
c) 455
d) -455

108. For natural numbers m, n if $(1-y)^m (1+y)^n = 1 + a_1 y^2 + \dots$and $a_1 = a_2 = 10$, then $(m, n)$is

a) (35,20)
b) (45,35)
c) (35,45)
d) (20,45)

109. If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4}$ is equal to

a) $\frac{a_2}{a_2+a_3}$
b) $\frac{1}{2}\frac{a_2}{(a_2+a_3)}$
c) $\frac{2a_2}{a_2+a_3}$
d) $\frac{2a_3}{a_2+a_3}$

110. If the sum of the coefficient in the expansion of $(a^2x^2 - 6ax + 11)^{10}$, where a is constant is 1024, then the value of a is

a) 5 b) 1 c) 2 d) 3

111. If x^{2r} occurs in $\left(x + \frac{2}{x^2}\right)^n$, then $n - 2r$ must be of the form

a) $3k - 1$ b) $3k$ c) $3k + 1$ d) $3k + 2$

112. $(2^{3n} - 1)$ will be divisible by $(\forall n \in N)$

a) 25 b) 8 c) 7 d) 3

113. If the sum of the coefficients in the expansion of $(\alpha\, x^2 - 2x + 1)^{35}$ is equal to the sum of the coefficient in the expansion of $(x - \alpha\, y)^{35}$, then $\alpha =$

a) 0 b) 1 c) Any real number d) None of these

114. If the ninth term in the expansion of $\left\{3^{\log_3 \sqrt{25^{x-1}+7}} + 3^{-1/8 \log_3 (5^{x-1}+1)}\right\}^{10}$ is equal to 180 and $x > 1$, then x equals

a) $\log_{10} 15$ b) $\log_5 15$ c) $\log_e 15$ d) None of these

115. The coefficient of x^{53} in the following expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$ is

a) ${}^{100}C_{47}$ b) ${}^{100}C_{53}$ c) $-{}^{100}C_{53}$ d) $-{}^{100}C_{100}$

116. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals

a) $-\frac{5}{3}$ b) $\frac{10}{3}$ c) $-\frac{3}{10}$ d) $\frac{3}{5}$

117. The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ will be

a) $\frac{3}{2}$

b) $\frac{5}{4}$

c) $\frac{5}{2}$

d) None of these

118. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$, then the value of $a_2 + a_4 + \ldots + a_{12}$ Is

a) 31 b) 32 c) 64 d) 1024

119. If $(1 + x - 3x^2)^{10} = 1 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$, then $a_2 + a_4 + a_6 + \ldots + a_{20}$ is equal to

a) $\frac{3^{10}+1}{2}$ b) $\frac{3^9+1}{2}$ c) $\frac{3^{10}-1}{2}$ d) $\frac{3^9-1}{2}$

120. If $(1 + x)^{2n} = a_0 + a_1\, x + a_2\, x^2 + \cdots + a_{2n}x^{2n}$, then

$(a_0 - a_2 + a_4 - a_6 + \cdots - a_{2n})^2 + (a_1 - a_3 + a_5 - a_7 + \cdots + a_{2n-1})^2$ is equal to

a) 2^n b) 4^n c) 0 d) None of these

121. The coefficient of $a^5b^6c^7$ in the expansion of $(bc + ca + ab)^9$ is

a) 100 b) 120 c) 720 d) 1260

122. In the polynomial $(x - 1)(x - 2)(x - 3) \ldots (x - 100)$. The coefficient of x^{99} is

a) 5050 b) -5050 c) 100 d) 99

123. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$ for every value of θ, then

a) $b_0 = 1, b_1 = 3$ b) $b_0 = 0, b_1 = n$

c) $b_0 = -1, b_1 = n$ d) $b_0 = 0, b_1 = n^2 - 3n + 3$

124. In the expansion of $\left(x + \sqrt{x^2-1}\right)^6 + \left(x - \sqrt{x^2-1}\right)^6$, the number of terms, is

a) 7 b) 14 c) 6 d) 4

125. If n is odd, then $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots + (-1)^n C_n^2$ is equal to

a) 0 b) 1 c) ∞ d) $\frac{n!}{\left(\frac{n}{2}\right)^2!}$

126. If $(1-x+x^2)^n = a_0 + a_1x + ...a_{2n}x^{2n}$ then the value of $a_0 + a_2 + a_4 + ...+a_{2n}$ is

a) $3^n + \frac{1}{2}$ b) $3^n - \frac{1}{2}$ c) $\frac{3^n - 1}{2}$ d) $\frac{3^n + 1}{2}$

127. ${}^{15}C_0 \cdot {}^5C_5 + {}^{15}C_1 \cdot {}^5C_4 + {}^{15}C_2 \cdot {}^5C_3 + {}^{15}C_3 \cdot {}^5C_2 + {}^{15}C_4 \cdot {}^5C_1$ is equal to

a) $2^{20} - 2^5$ b) $\frac{20!}{5!\,15!} - 1$ c) $\frac{20!}{5!\,15!} - 1$ d) $\frac{20!}{5!\,15!} - \frac{15!}{5!\,10!}$

128. In the expansion of the following expression $1 + (1+x) + (1+x)^2 + \ldots + (1+x)^n$, the coefficient of $x^4 (0 \le k \le n)$ is

a) ${}^{n+1}C_{k+1}$ b) ${}^nC_k$ c) ${}^nC_{n-k-1}$ d) None of these

129. In the binomial expansion of $(a-b)^n, n \ge 5$, the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals

a) $\frac{5}{n-4}$ b) $\frac{6}{n-5}$ c) $\frac{n-5}{6}$ d) $\frac{n-4}{5}$

130. The middle term in the expansion of $\left(1-\frac{1}{x}\right)^n (1-x)^n$, is

a) ${}^{2n}C_n$ b) $-{}^{2n}C_n$ c) $-{}^{2n}C_{n-1}$ d) None of these

131. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$, the constant term, is

a) ${}^{15}C_6$ b) 0 c) $-{}^{15}C_6$ d) 1

132. The number of terms in the expansion of $(a+b+c)^{10}$ is

a) 11 b) 21 c) 55 d) 66

133. The expansion of $\frac{1}{(4-3x)^{1/2}}$ by Only One Correct Option will be valid, if

a) $x < 1$

b) $|x| < 1$

c) $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$

d) None of these

134. The largest term in the expansion of $(3+2x)^{50}$, where $x = \frac{1}{5}$ is

a) 5th b) 3rd c) 7th d) 6th

135. If $(1+ax)^n = 1 + 8x + 24x^2 + \ldots$, then the values of a and n are

a) 2, 4 b) 2, 3 c) 3, 6 d) 1, 2

136. The value of $(0.99)^{15}$ is

a) 0.8432 b) 0.8601 c) 0.8502 d) None of these

137. $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots$ is equal to

a) $\frac{2^n}{n!}$ b) $\frac{2^{n-1}}{n!}$ c) 0 d) None of these

138. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as

a) $\frac{x}{2} - \frac{3}{8}x^2$ b) $-\frac{3}{8}x^2$ c) $3x + \frac{3}{8}x^2$ d) $1 - \frac{3}{8}x^2$

139. The number of terms in the expansion of $(2x+3y-4z)^n$, is

a) $n+1$ b) $n+3$ c) $\frac{(n+1)(n+2)}{2}$ d) None of these

140. If m, n, r are positive integers such that $r < m, n$, then ${}^mC_r + {}^mC_{r-1}\,{}^nC_1 + {}^mC_{r-2}\,{}^nC_2 + \cdots + {}^mC_1\,{}^nC_{r-1} + {}^nC_r$ equals

a) $({}^nC_r)^2$ b) ${}^{m+n}C_r$ c) ${}^{m+n}C_r + {}^mC_r + {}^nC_r$ d) None of these

141. If the expansion in power of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$, then a_n is

a) $\frac{a_n - b^n}{b-a}$ b) $\frac{a^{n+1} - b^{n+1}}{b-a}$ c) $\frac{b^{n+1} - a^{n+1}}{b-a}$ d) $\frac{b^n - a^n}{b-a}$

142. If $(1+2x+x^2)^5 = \sum_{k=0}^{15} a_k x^k$, then $\sum_{k=0}^{7} = a_{2k}$ is equal to

a) 128 b) 156 c) 512 d) 1024

143. If n is even, then the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924x^6$, then n is equal to

a) 10 b) 12 c) 14 d) None of these

144. The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$ is

a) 30
b) 60
c) 40
d) None of these

145. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^n$ is

a) ${}^nC_4$
b) ${}^nC_4 + {}^nC_2$
c) ${}^nC_4 + {}^nC_2 + {}^nC_2$
d) ${}^nC_4 + {}^nC_2 + {}^nC_1 \cdot {}^nC_2$

146. If a, b, c, d be four consecutive coefficients in the binomial expansion of $(1+x)^n$, then the value of the expression $\left\{\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right\}$ (where $x > 0$) is

a) < 0 b) > 0 c) $= 0$ d) 2

147. The coefficient of x^3 in $\left(\sqrt{x^5} + \frac{3}{\sqrt{x^3}}\right)^6$, is

a) 0 b) 120 c) 420 d) 540

148. The coefficient of x^{-7} in the expansion of $\left[ax - \frac{1}{bx^2}\right]^{11}$ will be

a) $\frac{462a^6}{b^5}$ b) $\frac{462a^5}{b^6}$ c) $-\frac{462a^5}{b^6}$ d) $-\frac{462a^6}{b^5}$

149. The coefficient of x^5 in the expansion of $(x+3)^6$ is

a) 18 b) 6 c) 12 d) 10

150. For $r = 0, \ldots, 10$ let A_r, B_r and C_r denotes, respectively, the coefficient of x^r in the $(1+x)^{10}, (1+x)^{20}$, and $(1+x)^{30}$. Then

$$\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$$

is equal to

a) $B_{10} - C_{10}$ b) $A_{10}(B_{10}^2 - C_{10}A_{10})$
c) 0 d) $C_{10} - B_{10}$

151. If p and q be positive, then the coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ will be

a) Equal
b) Equal in magnitude but opposite in sign
c) Reciprocal to each other
d) None of the above

152. If for positive integers $r > 1, n > 2$, the coefficient of the $(3r)$th and $(r+2)$th powers of x in the expansion of $(1+x)^{2n}$ are equal, then

a) $n = 2r$ b) $n = 3r$ c) $n = 2r+1$ d) None of these

153. The range of values of the term independent of x in the expansion of $\left(x\sin^{-1}\alpha+\frac{\cos^{-1}\alpha}{x}\right)^{10}$, $\alpha\in[-1,1]$, is

a) $\left[-\frac{{}^{10}C_5\,\pi^{10}}{2^5},\frac{{}^{10}C_5\,\pi^{10}}{2^{20}}\right]$ b) $\left[\frac{{}^{10}C_5\,\pi^{2}}{2^{20}},\frac{{}^{10}C_5\,\pi^{2}}{2^{5}}\right]$ c) $[1,2]$ d) $(1,2)$

154. If the coefficient of rth and $(r+1)$th terms in the expansion of $(3+7x)^{29}$ are equal, then r equals

a) 15 b) 21 c) 14 d) None of these

155. If the third term in the expansion $\left[x+x^{\log_{10}x}\right]^5$ is 10^6, then $x(>1)$ may be

a) 1 b) 10 c) $10^{-5/2}$ d) 10^2

156. In the expansion of $(1+x)^{50}$, the sum of the coefficient of add power of x is

a) Zero b) 2^{49} c) 2^{50} d) 2^{51}

157. If the coefficients of r^{th} and $(r+1)^{th}$ terms in the expansion of $(3+7x)^{29}$ are equal, then $r=$

a) 15 b) 21 c) 14 d) None of these

158. In the expansion of $(1+x)^{2n}(n\in N)$, the coefficients of $(p+1)^{th}$ and $(p+3)^{th}$ terms are equal, then

a) $p=n-2$ b) $p=n-1$ c) $p=n+1$ d) $p=2n-2$

159. Let $(1+x)^n=\sum_{r=0}^{n}a_r\,x^r$. Then,

$\left(1+\frac{a_1}{a_0}\right)\left(1+\frac{a_2}{a_1}\right)\dots\left(1+\frac{a_n}{a_{n-1}}\right)$ *is equal to*

a) $\frac{(n+1)^{n+1}}{n\,!}$ b) $\frac{(n+1)^{n}}{n\,!}$ c) $\frac{n^{n-1}}{(n-1)\,!}$ d) $\frac{(n+1)^{n-1}}{(n-1)\,!}$

160. If $C_0, C_1, C_2, \dots, C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then the value of $\sum_{r=0}^{n}(r+1)C_r$, is

a) $n\,2^n$ b) $(n+1)2^{n-1}$ c) $(n+2)2^{n-1}$ d) $(n+2)2^{n-2}$

161. If $(2x^2-x-1)^5=a_0+a_1x+a_2x^2+\ldots+a_{10}x^{10}$, then $a_2+a_4+a_6+a_8+a_{10}$is equal to

a) 15 b) 30 c) 16 d) 32

162. If the coefficient of $(r+1)^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ be equal to that of $(r+3)^{th}$ term, then

a) $n-r+1=0$ b) $n-r-1=0$ c) $n+r+1=0$ d) None of these

163. The coefficient of x^{100} in the expansion of $\sum_{j=0}^{200}(1+x)^j$ is

a) $\binom{200}{100}$ b) $\binom{201}{102}$ c) $\binom{200}{101}$ d) $\binom{201}{100}$

164. The value of $\frac{1}{n\,!}+\frac{1}{2\,!(n-2)\,!}+\frac{1}{4\,!(n-4)\,!}+\cdots$, is

a) $\frac{2^{n-2}}{(n-1)\,!}$ b) $\frac{2^{n-1}}{n\,!}$ c) $\frac{2^n}{n\,!}$ d) $\frac{2^n}{(n-1)\,!}$

165. The coefficient of x^4 in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

a) $\frac{504}{259}$ b) $\frac{450}{263}$ c) $\frac{405}{256}$ d) None of these

166. If $(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n$.Then , $C_0C_1+C_1C_2+\ldots+C_{n-1}C_n$ is equal to

a) $\frac{(2n)!}{(n-1)!\,(n+1)!}$ b) $\frac{(2n-1)!}{(n-1)!\,(n+1)!}$ c) $\frac{2n!}{(n+2)!\,(n+1)!}$ d) None of these

167. 7^9+9^7 is divided by

a) 128 b) 24 c) 64 d) 72

168. If $n>\left(8+3\sqrt{7}\right)^{10}$, $n\in N$, then the last value of n is

a) $\left(8+3\sqrt{7}\right)^{10}-\left(8-3\sqrt{7}\right)^{10}$ b) $\left(8+3\sqrt{7}\right)^{10}+\left(8-3\sqrt{7}\right)^{10}$

c) $\left(8+3\sqrt{7}\right)^{10}-\left(8-3\sqrt{7}\right)^{10}+1$ d) $\left(8+3\sqrt{7}\right)^{10}-\left(8-3\sqrt{7}\right)^{10}-1$

169. The ninth term of the expansion $\left(3x-\frac{1}{2x}\right)^8$ is

a) $\frac{1}{512x^9}$ b) $\frac{-1}{512x^9}$ c) $\frac{-1}{256x^8}$ d) $\frac{1}{256x^8}$

170. If x^{2k} occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{n-3}$, then

a) $n-2k$ is a multiple of 2
b) $n-2k$ is a multiple of 3
c) $k=0$
d) None of the above

171. The number of terms with integral coefficients in the expansion of $\left(7^{1/3}+5^{1/2}x\right)^{600}$, is

a) 100 b) 50 c) 101 d) None of these

172. The coefficient of $x^3y^4z^5$ in the expansion of $(xy+yz+xz)^6$ is

a) 70 b) 60 c) 50 d) None of these

173. If $(1+2x+x^2)^n=\sum_{r=0}^{2n} a_r x^r$, then $a_r=$

a) $({}^nC_r)^2$ b) ${}^nC_r\cdot{}^nC_{r+1}$ c) ${}^{2n}C_r$ d) ${}^{2n}C_{r+1}$

174. ${}^{20}C_4+2\cdot{}^{20}C_3+{}^{20}C_2-{}^{22}C_{18}$ is equal to

a) 0 b) 1242 c) 7315 d) 6345

175. If $y=3x+6x^2+10x^3+\cdots$, then $x=$

a) $\frac{4}{3}-\frac{1\cdot4}{3^2\cdot2}y^2+\frac{1\cdot4\cdot7}{3^2\cdot3}y^3\ldots$

b) $-\frac{4}{3}+\frac{1\cdot4}{3^2\cdot2}y^2-\frac{1\cdot4\cdot7}{3^2\cdot3}y^3+\cdots$

c) $\frac{4}{3}+\frac{1\cdot4}{3^2\cdot2}y^2+\frac{1\cdot4\cdot7}{3^2\cdot3}y^3+\cdots$

d) None of these

176. The expression $\{x+(x^2-1)^{1/2}\}^5+\{x-(x^2-1)^{1/2}\}^5$ is a polynomial of degree

a) 5 b) 6 c) 7 d) 8

177. The value of $C_0^2+3\cdot C_1^2+5\cdot C_2^2+\cdots$ to $(n+1)$ terms, is

a) ${}^{2n-1}C_{n-1}$
b) $(2n+1)^{2n-1}C_n$
c) $2(n+1)\cdot{}^{2n-1}C_{n-1}$
d) ${}^{2n-1}C_n+(2n+1)\,{}^{2n-1}C_{n-1}$

178. If $n-{}^1C_r=(k^2-3)\,{}^nC_{r+1}$, then $k\in$

a) $(-\infty,-2)$ b) $[2,\infty)$ c) $[-\sqrt{3},\sqrt{3}]$ d) $(\sqrt{3},2]$

179. The total number of terms in the expansion of $(x+y)^{100}+(x-y)^{100}$ after simplification is

a) 51 b) 202 c) 100 d) 50

180. If $(1+x)^n=C_0+C_1x+C_2x^2+\cdots+C_nx^n$, thn for n odd, $C_1^2+C_3^2+C_5^2+\cdots+C_n^2$ is equal to

a) 2^{2n-2} b) 2^n c) $\frac{(2n)!}{2(n!)^2}$ d) $\frac{(2n)!}{(n!)^2}$

181. $\sum_{k=0}^{10}\ {}^{20}C_k$ is equal to

a) $2^{19}+\frac{1}{2}\,{}^{20}C_{10}$ b) 2^{19} c) ${}^{20}C_{10}$ d) None of these

182. The approximate value of $(7.995)^{1/3}$ correct to four decimal places is

a) 1.9995 b) 1.9996 c) 1.9990 d) 1.9991

183. If the binomial coefficients of 2nd, 3rd and 4th terms in the expansion of

$\left\{\sqrt{2^{\log_{10}(10-3^x)}}+\sqrt[5]{2^{(x-2)\log_{10}3}}\right\}^m$ are in A.P and the 6th term is 21, then the value(s) of x, is (are)

a) 1, 3 b) 0, 2 c) 4 d) −1

184. If ${}^nC_{12}={}^nC_6$, then ${}^nC_2$ is equal to

a) 72 b) 153 c) 306 d) 2556

185. In the expansion of $\left(x-\frac{1}{x}\right)^6$, the coefficient of x^0 is

a) 20 b) −20 c) 30 d) −30

186. The term independent of x in the expansion of

$\left(x^3+\frac{2}{x^2}\right)^{15}$ is

a) T_7 b) T_8 c) T_9 d) T_{10}

187. If the $(r+1)$th term in the expansion of $\left(\frac{a^{1/3}}{b^{1/6}}+\frac{b^{1/2}}{a^{1/6}}\right)^{21}$ has equal exponents of both a and b, then value of r is

a) 8 b) 9 c) 10 d) 11

188. The coefficient of $1/x$ in the expansion of $\left(\frac{1}{x}+1\right)^n(1+x)^n$ is

a) ${}^{2n}C_n$ b) ${}^{2n}C_{n-1}$ c) ${}^{2n}C_1$ d) ${}^{n}C_{n-1}$

189. Let $[x]$ denote the greatest integer less than or equal to x. If $x=\left(\sqrt{3}+1\right)^5$, then $[x]$ is equal to

a) 75 b) 50 c) 76 d) 152

190. The value of $2\,C_0+\frac{2^2}{2}C_1+\frac{2^3}{3}C_2+\frac{2^4}{4}C_3+\cdots+\frac{2^{11}}{11}C_{10}$, is

a) $\frac{3^{11}-1}{11}$ b) $\frac{2^{11}-1}{11}$ c) $\frac{11^3-1}{11}$ d) $\frac{11^2-1}{11}$

191. The sum of coefficients of the expansion $\left(\frac{1}{x}+2x\right)^n$ in 6561. The coefficient of term independent of x is

a) $16\ {}^{8}C_4$ b) ${}^{8}C_4$ c) ${}^{8}C_5$ d) None of these

192. In the expansion of $(1+x)^{30}$, the sum of the coefficients of odd powers of x is

a) 2^{30} b) 2^{31} c) 0 d) 2^{29}

193. The 6th term in the expansion of $\left(2x^2-\frac{1}{3x^2}\right)^{10}$ is

a) $\frac{4580}{17}$ b) $-\frac{896}{27}$ c) $\frac{5580}{17}$ d) None of these

194. ${}^{47}C_4+\sum_{r=1}^{5}\ {}^{52-r}C_3$ is equal to

a) ${}^{45}C_6$ b) ${}^{52}C_5$ c) ${}^{52}C_4$ d) None of these

195. The coefficient of x^n in the expansion of $(1-2\,x+3\,x^2-4\,x^3+\cdots)^{-n}$, is

a) $\frac{(2\,n)\,!}{n\,!}$ b) $\frac{(2\,n)\,!}{(n\,!)^2}$ c) $\frac{1}{2}\frac{(2\,n)\,!}{(n\,!)^2}$ d) None of these

196. The term independent of x in the expansion of $(1+x)^n(1+1/x)^n$, is

a) $C_0^2+2\,C_1^2+3\cdot C_2^2+\cdots+(n+1)C_n^2$

b) $(C_0+C_1+\cdots+C_n)^2$

c) $C_0^2+C_1^2+\cdots+C_n^2$

d) None of these

197. If A and B are coefficients of x^r and x^{n-r} respectively in the expansion of $(1+x)^n$, then

a) $A=B$ b) $A+B=0$ c) $A=r\,B$ d) $A=nB$

198. If $x=\frac{\left[\begin{matrix}729+6(2)(243)+15(4)(81)\\+20(8)(27)+15(16)(9)\\+6(32)3+64\end{matrix}\right]}{1+4(4)+6(16)+4(64)+256}$, then $\sqrt{x}-\frac{1}{\sqrt{x}}$ isequal to

a) 0.2 b) 4.8 c) 1.02 d) 5.2

199. If the coefficients of pth, $(p+1)$th and $(p+2)$th terms in the expansion of $(1+x)^n$ are in AP, then

a) $n^2-2np+4p^2=0$

b) $n^2-n(4p+1)+4p^2-2=0$

c) $n^2-n(4p+1)+4p^2=0$

d) None of the above

200. The sum of the rational terms in the expansion of $(\sqrt{2}+3^{1/5})^{10}$ is

a) 41 b) 32 c) 18 d) 9

201. Let T_n denotes the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals

a) 5 b) 7 c) 6 d) 4

202. Sum of the last 30 coefficients in the expansion of $(1+x)^{59}$, when expanded in ascending powers of x is

a) 2^{59} b) 2^{58} c) 2^{30} d) 2^{29}

203. If $|x| < 1$, then $1 + n\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \ldots$ is equal to

a) $\left(\frac{2x}{1+x}\right)^n$ b) $\left(\frac{1+x}{2x}\right)^n$ c) $\left(\frac{1-x}{1+x}\right)^n$ d) $\left(\frac{1+x}{1-x}\right)^n$

204. In the expansion of $(1+3x+2x^2)^6$ the coefficient of x^{11} is

a) 144 b) 288 c) 216 d) 576

205. The term independent of x in the expansion of $(1-x)^2\left(x+\frac{1}{x}\right)^{10}$, is

a) ${}^{11}C_5$ b) ${}^{10}C_5$ c) ${}^{10}C_4$ d) None of these

206. If the sum of the coefficient in the expansion of $(x-2y+3z)^n$ is 128, then the greatest coefficient in the expansion of $(1+x)^n$ is

a) 35 b) 20 c) 10 d) None of these

207. The digit at the unit place in the number $19^{2005} + 11^{2005} - 9^{2005}$ is

a) 2 b) 1 c) 0 d) 8

208. The sum of the series

$\sum_{r=0}^{n}(-1)^r\, {}^nC_r\left(\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \ldots + m \text{ terms}\right)$ is

a) $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$

b) $\frac{2^{mn}-1}{2^n-1}$

c) $\frac{2^{mn}+1}{2^n+1}$

d) None of these

209. r and n are positive integers such that $r > 1, n > 2$ and coefficients of $(r+2)^{\text{th}}$ term and $(3r)^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals

a) $3r$ b) $3r+1$ c) $2r$ d) $2r+1$

210. Assuming to be small that x^2 and higher powers of x can be neglected, then $\frac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}}$ is approximately equal to

a) $1+\frac{305}{96}x$ b) $1-\frac{305}{96}x$ c) $1+\frac{96}{305}x$ d) $1-\frac{96}{305}x$

211. The coefficient of x^5 in $(1+x^2)^5(1+x)^4$ is

a) 20 b) 30 c) 60 d) 55

212. The number of terms in the expansion of $\left(1+5\sqrt{2}\,x\right)^9 + \left(1-5\sqrt{2}\,x\right)^9$, is

a) 5 b) 7 c) 9 d) 10

213. If $x = \frac{1}{3}$, then the greatest term in the expansion of $(1+4x)^8$ is the

a) 3rd term b) 6th term c) 5th term d) 4th term

214. The coefficient of x^n in the expansion of $(1+2x+3x^2+\ldots)^{1/2}$ is

a) -1 b) 0 c) 2 d) 1

215. The coefficient of $\lambda^n\mu^n$ in the expansion of $[(1+\lambda)(1+\mu)(\lambda+\mu)]^n$ is

a) $\sum_{r=0}^{n} C_r^2$ b) $\sum_{r=0}^{n} C_{r+2}^2$ c) $\sum_{r=0}^{n} C_{r+3}^2$ d) $\sum_{r=0}^{n} C_r^3$

216. If $(1+x-2x^2)^6 = 1 + C_1x + C_2x^2 + C_3x^3 + \cdots + C_{12}x^{12}$, then the value of $C_2 + C_4 + C_6 + \cdots + C_{12}$, is

a) 30 b) 32 c) 31 d) None of these

217. If in the expansion of $(1+ax)^n, n \in N$, the coefficients of x and x^2 are 8 and 24 respectively, then
a) $a=2, n=4$ b) $a=4, n=2$ c) $a=2, n=6$ d) $a=-2, n=4$

218. The greatest coefficient in the expansion of $(1+x)^{2n}$ is
a) ${}^{2n}C_n$ b) ${}^{2n}C_{n+1}$ c) ${}^{2n}C_{n-1}$ d) ${}^{2n}C_{2n-1}$

219. The coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is
a) $\frac{405}{256}$ b) $\frac{504}{259}$ c) $\frac{450}{263}$ d) None of these

220. If n is an odd natural number and ${}^nC_0 < {}^nC_1 < {}^nC_2 < \cdots < {}^nC_r > {}^nC_{r+1} > {}^nC_{r+2} > \cdots > {}^nC_n$, then $r=$
a) $\frac{n}{2}$ b) $\frac{n-1}{2}$ c) $\frac{n-2}{2}$ d) Does not exist

221. If $[x]$ denotes the greatest integer less than or equal to x, and $F=R-[R]$ where $R=(5\sqrt{5}+11)^{2n+1}$, then $R\,F$ is equal to
a) 4^{2n+1} b) 4^{2n} c) 4^{2n-1} d) None of these

222. If the coefficients of 5th, 6th and 7th terms in the expansion of $(1+x)^n$ be in AP, then the value of n is
a) 7 only b) 14 only c) 7 or 14 d) None of these

223. Let $R=(2+\sqrt{3})^{2n}$ and $f=R-[R]$ where $[\cdot]$ denotes the greatest integer function, then $R(1-f)$ is equal to
a) 1 b) 2^{2n} c) $2^{2n}-1$ d) ${}^{2n}C_n$

224. The value of
$({}^7C_0+{}^7C_1)+({}^7C_1+{}^7C_2)+\cdots+({}^7C_6+{}^7C_7)$ is
a) 2^8-1 b) 2^8+1 c) 2^8 d) 2^8-2

225. The value of ${}^{4n}C_0+{}^{4n}C_4+{}^{4n}C_8+\ldots+{}^{4n}C_{4n}$ is
a) $2^{4n-2}+(-1)^n 2^{2n-1}$
b) $2^{4n-2}+2^{2n-1}$
c) $2^{2n-1}+(-1)^n 2^{4n-2}$
d) None of these

226. If there is a term containing x^{2r} in $\left(x+\frac{1}{x^2}\right)^{n-3}$, then
a) $n-2r$ is a positive integral multiple of 3
b) $n-2r$ is even
c) $n-2r$ is odd
d) None of these

227. Last two digit of the number 19^{9^4} is
a) 19 b) 29 c) 39 d) 81

228. $1+\frac{1}{3}x+\frac{1\cdot 4}{3\cdot 6}x^2+\frac{1\cdot 4\cdot 7}{3\cdot 6\cdot 9}x^3+\ldots$ is equal to
a) x b) $(1+x)^{1/3}$ c) $(1-x)^{1/3}$ d) $(1-x)^{-1/3}$

229. What is the sum of the coefficient of $(x^2-x-1)^{99}$?
a) 1 b) 0 c) -1 d) None of these

230. If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also, is
a) $\frac{n}{n+2}<x<\frac{n+2}{n}$ b) $\frac{n+1}{n}<x<\frac{n}{n+1}$ c) $\frac{n}{n+4}<x<\frac{n+4}{4}$ d) None of these

231. The value of ${}^{15}C_8+{}^{15}C_9-{}^{15}C_6-{}^{15}C_7$ is
a) -1 b) 0 c) 1 d) None of these

232. If $(1-x+x^2)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$, then the $a_0+a_2+a_4+\ldots+a_{2n}$ is equal to
a) $\frac{3^n+1}{2}$
b) $\frac{3^n-1}{2}$

c) $\frac{1-3^n}{2}$

d) $3^n+\frac{1}{2}$

233. If a_r is the coefficient of x^r , in the expansion of $(1+x+x^2)^n$, then $a_1-2a_2+3a_3-\ldots-2n\,a_{2n}$ is equal to

a) 0　b) n　c) $-n$　d) $2n$

234. If ${}^{n-1}C_r=(k^2-3).\,{}^{n}C_{r+1}$, then k is belongs to

a) $(-\infty,-2]$　b) $[2,\infty)$　c) $[-\sqrt{3},\sqrt{3}]$　d) $(\sqrt{3},2]$

235. The greatest value of the term independent of x, as α varies over R, in the expansion of $\left(x\cos\alpha+\frac{\sin\alpha}{x}\right)^{20}$ is

a) ${}^{20}C_{10}$　b) ${}^{20}C_{15}$　c) ${}^{20}C_{19}$　d) None of these

236. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is p and the coefficients of middle term in the expansion of $(1+x)^{2n+1}$ are q and r, then

a) $p+q=r$　b) $p+r=q$　c) $p=q+r$　d) $p+q+r=0$

237. If sum of the coefficients of the first, second and third terms of the expansion of $\left(x^2+\frac{1}{x}\right)^m$ is 46, then the coefficient of the term that does not contain x is

a) 84

b) 92

c) 98

d) 106

238. If the r^{th} $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ coefficients of $(1+x)^n$ are in A.P., then n is a root of the equation

a) $x^2-x(4r+1)+4r^2-2=0$

b) $x^2+x(4r+1)+4r^2-2=0$

c) $x^2+x(4r+1)+4r^2+2=0$

d) None of these

239. If $|x|<1$, then the coefficient of x^6 in the expansion of $(1+x+x^2)^{-3}$ is

a) 3　b) 6　c) 9　d) 12

240. $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$ is equal to

a) $\frac{n+1}{n!}$　b) $\frac{(n+1)^n}{(n-1)!}$　c) $\frac{(n-1)^n}{n!}$　d) $\frac{(n+1)^n}{n!}$

241. The coefficient of x^{20} in the expansion of $(1+3x+3x^2+x^3)^{20}$ is

a) ${}^{60}C_{40}$　b) ${}^{30}C_{20}$　c) ${}^{15}C_2$　d) None of these

242. The greatest value of the term independent of x in the expansion of $(x\sin\alpha+x^{-1}\cos\alpha)^{10}, \alpha\in R$ is

a) 2^5　b) ${}^{10}C_5$　c) $\frac{1}{2^5}\,({}^{10}C_5)$　d) None of these

243. If $(1+2x+3x^2)^{10}=a_0+a_1\,x+a_2\,x^2+\cdots+a_{20}\,x^{20}$, then a_1 equals

a) 10　b) 20　c) 210　d) None of these

244. If the coefficient of rth, $(r+1)$th and $(r+2)$ th terms in the binomial expansion of $(1+y)^m$ are in AP, then m and r satisfy the equation

a) $m^2-m(4r-1)+4r^2+2=0$　b) $m^2-m(4r+1)+4r^2-2=0$

c) $m^2-m(4r+1)+4r^2+2=0$　d) $m^2-n(4r-1)+4r^2-2=0$

245. Coefficient of the term independent of x in the expansion

$\left(x+\frac{1}{x^2}\right)^6$ is equal to

a) 10　b) 15　c) 16　d) None of the above

246. The last term in the binomial expansion of $\left(\sqrt[3]{2}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$. Then, the 5^{th} term from the beginning is

a) ${}^{10}C_6$ b) $2 \times {}^{10}C_4$ c) $\frac{1}{2} \times {}^{10}C_4$ d) None of these

247. Using mathematical induction, then numbers $a_n's$ are defined by $a_0=1, a_{n+1} = 3n^2 + n + a_n, (n \geq 0)$ Then, a_n is equal to

a) $n^3 + n^2 + 1$ b) $n^3 - n^2 + 1$ c) $n^3 - n^2$ d) $n^3 + n^2$

248. The coefficient of x^{-10} in $\left(x^2 - \frac{1}{x^3}\right)^{10}$ is

a) -252 b) 210 c) $-(5!)$ d) -120

249. The value of $C_0 + 3\,C_1 + 5\,C_2 + 7\,C_3 \ldots + (2\,n+1)\,C_n$ is equal to

a) 2^n b) $2^n + n \cdot 2^{n-1}$ c) $2^n \cdot (n+1)$ d) None of these

250. If p is nearly equal to q and $n > 1$, such that $\frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q} = \left(\frac{p}{q}\right)^k$, then the value of k, is

a) n b) $\frac{1}{n}$ c) $n+1$ d) $\frac{1}{n+1}$

251. If the sum of the coefficients in the expansion of $(a^2x^2 - 2ax + 1)^{51}$ vanishes, then the value of a is

a) 2 b) -1 c) 1 d) -2

252. $1 + \frac{2}{4} + \frac{2 \cdot 5}{4 \cdot 8} + \frac{2 \cdot 5 \cdot 8}{4 \cdot 8 \cdot 12} + \frac{2 \cdot 5 \cdot 8 \cdot 11}{4 \cdot 8 \cdot 12 \cdot 16} + \ldots$ is

a) $4^{-2/3}$ b) $\sqrt[3]{16}$ c) $\sqrt[3]{4}$ d) $4^{3/2}$

253. The coefficient of $x^r (0 \leq r \leq (n-1))$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}\,(x+2)^2 + \cdots + (x+2)^{n-1}$, is

a) ${}^nC_r(3^r - 2^n)$ b) ${}^nC_r(3^{n-r} - 2^{n-r})$ c) ${}^nC_r(3^r + 2^{n-r})$ d) None of these

254. Let C_1, C_2, C_3 are the usual binomial coefficients. Let $S = C_1 + 2C_2 + 3C_3 + \ldots + nC_n$, then S equals

a) $n2^n$ b) 2^{n-1} c) $n2^{n-1}$ d) 2^{n+1}

255. If the value of x is so small that x^2 and greater powers can be neglected, then $\frac{\sqrt{1+x}+\sqrt[3]{(1-x)^2}}{1+x+\sqrt{1+x}}$ is equal to

a) $1 + \frac{5}{6}x$ b) $1 - \frac{5}{6}x$ c) $1 + \frac{2}{3}x$ d) $1 - \frac{2}{3}x$

256. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is

a) $(n-1)$ b) $(-1)^n(1-n)$ c) $(-1)^{n-1}(n-1)^2$ d) $(-1)^{n-1}n$

257. The middle term in the expansion of $(x-a)^8$ is

a) $-{}^8C_4x^4a^4$ b) ${}^8C_4x^4a^4$ c) ${}^8C_3x^5a^3$ d) $-{}^8C_5x^2a^5$

258. If the expansion of $(1+x)^{20}$, the coefficients of r^{th} and $(r+4)^{th}$ terms are equal, then the value of r, is

a) 7 b) 8 c) 9 d) 10

259. If $(1+x-2x^2)^6 = 1 + a_1\,x + a_2\,x^2 + \cdots + a_{12}\,x^{12}$, then $a_2 + a_4 + a_6 + \cdots + a_{12} =$

a) 30 b) 65 c) 31 d) 63

260. The value of $\frac{1}{81^n} = \frac{10}{81^n}\,{}^{2n}C_1 + \frac{10^2}{81^n}\,{}^{2n}C_2 - \frac{10^3}{81^n}\,{}^{2n}C_3 + \cdots + \frac{10^{2n}}{81^n}$, is

a) 2 b) 0 c) 1/2 d) 1

261. The value of $\left(\frac{{}^{50}C_0}{1} + \frac{{}^{50}C_2}{3} + \frac{{}^{50}C_4}{5} + \ldots + \frac{{}^{50}C_{50}}{51}\right)$ is

a) $\frac{2^{50}}{51}$ b) $\frac{2^{50}-1}{51}$ c) $\frac{2^{50}-1}{50}$ d) $\frac{2^{51}-1}{51}$

262. Matrix A is such that $A^2 = 2A - I$ where I is the identity matrix, then for $n \geq 2$, A^n is equal to

a) $n\,A - (n-1)I$ b) $n\,A - I$ c) $2^{n-1}A - (n-1)I$ d) $2^{n-1}A - I$

263. If $\sum_{r=0}^{2n} a_r(x-100)^r = \sum_{r=0}^{2n} b_r(x-101)^r$ and $a_k = \frac{2^k}{{}^kC_n}$ for all $k \geq n$, then b_n equals

a) $2^n(2^{n+1}-1)$ b) $2^n(2^n+1)$ c) $2^n(2^n-1)$ d) $2^{n+1}(2^n-1)$

264. The coefficient of $x^r [0 \le r \le (n-1)]$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots + (x+2)^{n-1}$ is

a) ${}^nC_r(3^r - 2^n)$
b) ${}^nC_r(3^{n-r} - 2^{n-r})$
c) ${}^nC_r(3^r + 2^{n-1})$
d) None of these

265. The middle term in the expansion of $\left(x + \frac{1}{x}\right)^{10}$, is

a) ${}^{10}C_1 \dfrac{1}{x}$ b) ${}^{10}C_5$ c) ${}^{10}C_6$ d) ${}^{10}C_7 x$

266. The sum of the magnitudes of the coefficients in the expansion of $(1 - x + x^2 - x^3)^n$, is

a) 0 b) 2^n c) 3^n d) 4^n

267. The coefficient of x^7 in the expansion of $(x - 2x^2)^{-3}$, is

a) 67485 b) 67548 c) 67584 d) 67845

268. If $n > 3$, then
$xyz\, C_0 - (x-1)(y-1)(z-1)C_1 + (x-2)(y-2)(z-2)C_2$
$-(x-3)(y-3)(z-3)C_3 + \cdots + (-1)^n (x-n)(y-n)(z-n)C_n$ equals

a) xyz b) $nxyz$ c) $-xyz$ d) 0

269. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + 15\frac{C_{15}}{C_{14}}$ is equal to

a) 100 b) 120 c) -120 d) None of these

270. The digit at unit's place in the number $17^{1995} + 11^{1995} - 7^{1995}$, is

a) 0 b) 1 c) 2 d) 3

271. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

a) ${}^{56}C_4$ b) ${}^{56}C_3$ c) ${}^{55}C_3$ d) ${}^{55}C_4$

272. The term independent of x in the expansion of
$\left(x - \frac{1}{x}\right)^4 \left(x + \frac{1}{x}\right)^3$ is

a) -3 b) 0 c) 3 d) 1

273. The coefficients of x^2y^2, yzt^2 and $xyzt$ in the expansion of $(x+y+z+t)^4$ are in the ratio

a) 4: 2: 1 b) 1: 2: 4 c) 2: 4: 1 d) 1: 4: 2

274. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$, then a_1 equals

a) 10 b) 20 c) 210 d) None of these

275. The coefficient of x^5 in the expansion of
$(1 + x^2)^5 (1 + x)^4$ is

a) 30 b) 60 c) 40 d) None of these

276. The sum of the series $\sum_{r=0}^{10} {}^{20}C_r$, is

a) 2^{20} b) 2^{19} c) $2^{19} + \dfrac{1}{2}\,{}^{20}C_{10}$ d) $2^{19} - \dfrac{1}{2}\,{}^{20}C_{10}$

277. If the last term in the binomial expansion of $\left(2^{1/3} - \frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}$, then the 5th term from the beginning is

a) 210 b) 420 c) 105 d) None of these

278. The coefficient of x^{-10} in $\left(x^2 - \frac{1}{x^3}\right)^{10}$, is

a) -252 b) 210 c) $-5!$ d) -120

279. The coefficient of x^n in the binomial expansion of $(1-x)^{-2}$, is

a) $\dfrac{2^n}{2!}$ b) $n+1$ c) n d) $2n$

280. Let $(1+x)^n = \sum_{r=0}^{n} C_r\, x^r$ and, $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \cdots + n\frac{C_n}{C_{n-1}} = \frac{1}{k} n(n+1)$, then the value of k, is

a) 1/2 b) 2 c) 1/3 d) 3

281. The coefficient of x^m in $(1+x)^p+(1+x)^{p+1}+\cdots+(1+x)^n, p\le m\le n$ is

a) ${}^{n+1}C_{m+1}$ b) ${}^{n-1}C_{m-1}$ c) ${}^{n}C_{m}$ d) ${}^{n}C_{m+1}$

282. If $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2+\ldots+{}^nC_nx^n$, then $\frac{{}^nC_1}{{}^nC_0}+\frac{2\,{}^nC_2}{{}^nC_1}+\frac{3\,{}^nC_3}{{}^nC_2}+\ldots+\frac{n\,{}^nC_n}{{}^nC_{n-1}}$ is equal to

a) $\frac{n(n-1)}{2}$

b) $\frac{n(n+2)}{2}$

c) $\frac{n(n+1)}{2}$

d) $\frac{(n-1)(n-2)}{2}$

283. If the magnitude of the coefficient of x^7 in the expansion of $\left(x^2+\frac{1}{bx}\right)^8$, where a, b, are positive numbers, is equal to the magnitude of the coefficient of x^{-7} in the of $\left(ax-\frac{1}{bx^2}\right)^8$, then a, and b are connected by the relation

a) $ab=1$ b) $ab=2$ c) $a^2b=1$ d) $ab^2=2$

284. If $T_0, T_1, T_2, \ldots, T_n$ represent the term in the expansion of $(x+a)^n$, then the value of $(T_0-T_2+T_4-T_6\ldots)^2+(T_1-T_3+T_5+\cdots)^2$, is

a) $(x^2-a^2)^n$ b) $(x^2+a^2)^n$ c) $(a^2-x^2)^n$ d) None of these

285. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, $\left(\text{where, }\binom{p}{q}=0 \text{ if } p<q\right)$ is maximum, when m is

a) 5 b) 10 c) 15 d) 20

286. Let $(1+x)^n = 1+a_1x+a_2x^2+\ldots+a_nx^n$. If a_1, a_2 and a_3 are in AP, then the value of n is

a) 4 b) 5 c) 6 d) 7

287. If $(1+x)^{15}=a_0+a_1x+\ldots+a_{15}x^{15}$, then $\sum_{r=1}^{15} r\frac{a_r}{a_{r-1}}$ is equal to

a) 110 b) 115 c) 120 d) 135

288. The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$, is

a) 30 b) 60 c) 40 d) None of these

289. The coefficient of x^r in the expansion of $(1-x)^{-2}$ is

a) r b) $r+1$ c) $r+3$ d) $r-1$

290. The expression $\frac{1}{\sqrt[3]{6-3x}}$ is equal to

a) $6^{1/3}\left[1+\frac{x}{6}+\frac{2x^2}{6^2}+\ldots\right]$

b) $6^{-1/3}\left[1+\frac{x}{6}+\frac{2x^2}{6^2}+\ldots\right]$

c) $6^{1/3}\left[1-\frac{x}{6}+\frac{2x^2}{6^2}-\ldots\right]$

d) $6^{-1/3}\left[1-\frac{x}{6}+\frac{2x^2}{6^2}-\ldots\right]$

291. If the coefficient of r^{th}, $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the expansion of $(1+x)^{14}$ are in A.P., then the value of r, is

a) 5,9 b) 6,9 c) 7,9 d) None of these

292. The interval in which x must lie so that the numerically greatest term in the expansion of $(1-x)^{21}$ has the numerically greatest coefficient, is

a) $\left[\frac{5}{6},\frac{6}{5}\right]$ b) $\left(\frac{5}{6},\frac{6}{5}\right)$ c) $\left(\frac{4}{5},\frac{5}{4}\right)$ d) $\left[\frac{4}{5},\frac{5}{4}\right]$

293. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}}+x^2\log_{10}x\right)^8$ is 5600, then value of x is

a) 2 b) $\sqrt{5}$ c) $\sqrt{10}$ d) 10

294. The coefficient of x^{20} in the expansion of $(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$, is

a) ${}^{30}C_{10}$ b) ${}^{30}C_{25}$ c) 1 d) None of these

295. If n is a positive integer, then n^3+2n is divisible by

a) 2 b) 6 c) 15 d) 3

296. If in the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and -6 respectively, then m is

a) 6 b) 9 c) 12 d) 24

297. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^n$, is

a) ${}^nC_4$

b) ${}^nC_4+{}^nC_2$

c) ${}^nC_4+{}^nC_1+{}^nC_4\times{}^nC_2$

d) ${}^nC_4+{}^nC_2+{}^nC_1\times{}^nC_2$

298. Sum of the infinite series

$1+\frac{1}{3}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{8}{9}\cdot\frac{1}{2^3}+\cdots\infty$ is

a) $2^{1/3}$ b) $4^{1/3}$ c) $8^{1/3}$ d) $2^{1/5}$

299. If in the expansion of $(a-2b)^n$.The sum of the 5th and 6th term is zero, then the value of $\frac{a}{b}$ is

a) $\frac{n-4}{5}$ b) $\frac{2(n-4)}{5}$ c) $\frac{5}{n-4}$ d) $\frac{5}{2(n-4)}$

300. If in the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and -6 respectively, then m is

a) 6 b) 9 c) 12 d) 24

301. If the sum of the coefficient in the expansion of $(x+y)^n$ is 1024, then the value of the greatest coefficient in the expansion is

a) 356 b) 252 c) 210 d) 120

302. The remainder when 5^{99} is divided by 13, is

a) 6 b) 8 c) 9 d) 10

303. The two consecutive terms in the expansion of $(3+2x)^{74}$ whose coefficient are equal, are

a) 11.12 b) 7,8 c) 30,31 d) None of these

304. If the 4th term in the expansion of $\left(\frac{2}{3}x-\frac{3}{2x}\right)^n$ is independent of x, then n is equal to

a) 5 b) 6 c) 9 d) None of these

305. The coefficient of x^{32} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is

a) ${}^{-15}C_3$ b) ${}^{15}C_4$ c) ${}^{-15}C_5$ d) ${}^{15}C_2$

306. The number of dissimilar terms in the expansion of $(a+b)^n$is $n+1$therefore no of dissimilar terms of the expansion $(a+b+c)^{12}$ is

a) 13 b) 39 c) 78 d) 91

307. The coefficient of x^7 in $(1+3x-2x^3)^{10}$ is equal to

a) 62640 b) 26240 c) 64620 d) None of these

308. The middel term in the expansion of $\left(x-\frac{1}{x}\right)^{18}$ is

a) ${}^{18}C_9$ b) $-{}^{18}C_9$ c) ${}^{18}C_{10}$ d) $-{}^{18}C_{10}$

309. The term independent of x in the expansion of

$\left(\frac{2\sqrt{x}}{5}-\frac{1}{2x\sqrt{x}}\right)^{11}$ is

a) 5th term b) 6th term c) 11th term d) No term

310. If n is an integer greater than 1, then

$a - {}^nC_1(a-1) + {}^nC_2(a-2) + \ldots + (-1)^n(a-n)$ is equal to

a) a b) 0 c) a^2 d) 2^n

311. The sum of the rational terms in the expansion of $\left(2^{1/5} + \sqrt{3}\right)^{20}$, is

a) 71 b) 85 c) 97 d) None of these

312. If $[x]$ denotes the gretest integer less than or equal to x, then $\left[\left(6\sqrt{6} + 14\right)^{2n+1}\right]$

a) Is an even integer b) Is an odd integer c) Depends on n d) None of these

313. If $n \in N$, then the sum of the coefficients in the expansion of the binomial $(5x - 4y)^n$, is

a) 1 b) -1 c) n d) 0

314. The coefficient of the term independent of x in the expansion of

$$\left[\frac{(x+1)}{x^{2/3} - x^{1/3} + 1} - \frac{(x-1)}{x - x^{1/2}}\right]^{10} \text{ is}$$

a) 210 b) 105 c) 70 d) 112

315. $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \frac{C_6}{7} + \ldots$ is equal to

a) $\frac{2^{n+1}}{n+1}$ b) $\frac{2^{n+1}-1}{n+1}$ c) $\frac{2^n}{n+1}$ d) None of these

316. Coefficient of x^n in the expansion of $\frac{(1+x)^n}{1-x}$

a) $4n$ b) 2^n c) n^2 d) $\frac{n(n+1)}{2}$

317. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$, then the expression $a_2 + a_4 + a_6 + \ldots + a_{12}$ has the value

a) 32 b) 63 c) 64 d) None of these

318. If $(1+x)^n = \sum_{r=0}^{n} a_r x^r$ and $b_r = 1 + \frac{a_r}{a_{r-1}}$ and $\prod_{r=1}^{n} b_r = \frac{(101)^{100}}{100!}$, then n is

a) 99 b) 100 c) 101 d) 102

319. Coefficient of $x^2y^3z^4$ in $(ax + by + cz)^9$ is

a) $1060a^2b^3c^4$ b) $1160a^2b^3c^4$ c) $1260a^2b^3c^4$ d) $960a^2b^3c^4$

320. The constant term in the expansion of

$$\left(x^2 - \frac{1}{x}\right)^9 \text{ is}$$

a) 80 b) 72 c) 84 d) 82

321. $$\sum_{k=1}^{\infty} k\left(1 + \frac{1}{n}\right)^{k-1} =$$

a) $n(n-1)$ b) $n(n+1)$ c) n^2 d) $(n+1)^2$

322. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1: 7: 42, then the value of n is

a) 60 b) 70 c) 55 d) None of these

323. The number of non-zero terms in the expansion of $\left(1 + 3\sqrt{2}\,x\right)^9 + \left(1 - 3\sqrt{2}\,x\right)^9$, is

a) 9 b) 0 c) 5 d) 10

324. If the coefficient of 7th and 13th term in the expansion of $(1+x)^n$ are equal, then n is equal to

a) 10 b) 15 c) 18 d) 20

325. In the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$, the term independent of x is

a) 8th b) 7th c) 9th d) 10th

326. If $C_0, C_1, C_2, \ldots, C_n$ are coefficients in the binomial expansion of $(1+x)^n$, then $C_0C_2 + C_1C_3 + C_2C_4 + \cdots + C_{n-2}C_n$ is equal to

a) $\frac{(2n)!}{(n-2)!\,(n+2)!}$ b) $\frac{(2n)!}{\left((n-2)!\right)^2}$ c) $\frac{(2n)!}{\left((n+2)!\right)^2}$ d) None of these

327. For all integers $n \geq 1$, which of the following is divisible by 9?

a) $8^n + 1$ b) $4^n - 3n - 1$ c) $3^{2n} + 3n + 1$ d) $10^n + 1$

328. The sum of the series
$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots + {}^{20}C_{10}$ is

a) $-{}^{20}C_{10}$ b) $\frac{1}{2}\,{}^{20}C_{10}$ c) 0 d) $^{20}C_{10}$

329. Let $[x]$ donate the greatest integer less than or equal to x. If $x = (\sqrt{3}+1)^5$, then $[x]$ is equal to

a) 75 b) 50 c) 76 d) 152

330. The coefficient of x^{53} in the expansion of

$$\sum_{m=0}^{100} {}^{100}C_m\,(x-3)^{100-m} \cdot 2^m$$

is

a) $^{100}C_{47}$ b) $^{100}C_{53}$ c) $^{-100}C_{53}$ d) $^{-100}C_{100}$

331. If the fourth term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, then

a) $a = 1/2$ and n = 6 b) $a = 1/3$ and n = 5 c) $a = 2$ and n = 3 d) $a = 1/4$ and n = 1

332. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, then
$\sum_{0 \le r < s \le n} \sum (r+s)C_rC_s$ is equal to

a) $n[2^{2n-1} - {}^{2n-1}C_{n-1}]$
b) $n[2^{2n-1} + {}^{2n-1}C_{n-1}]$
c) $2n[2^{2n-1} - {}^{2n-1}C_{n-1}]$
d) None of these

333. If the coefficient of x^7 in the expansion of $(ax^2 + b^{-1}x^{-1})^{11}$ is equal to the coefficient of x^{-7} in $(ax - b^{-1}x^{-2})^{11}$, then $ab =$

a) 1 b) 2 c) 3 d) 4

334. The coefficient of x^n in the expansion of $\frac{1}{(1-x)(3-x)}$, is

a) $\frac{3^{n+1}-1}{2 \cdot 3^{n+1}}$ b) $\frac{3^{n+1}-1}{3^{n+1}}$ c) $2\left(\frac{3^{n+1}-1}{3^{n+1}}\right)$ d) None of these

335. The greatest term in the expansion of $(1+3x)^{54}$, where $x = 1/3$ is

a) T_{28} b) T_{25} c) T_{26} d) T_{24}

336. If in the expansion of $\left(\frac{1}{x} + x \tan x\right)^5$ the ratio of 4th term to the 2nd term is $\frac{2}{27}\pi^4$, then the value of x can be

a) $-\frac{\pi}{6}$ b) $-\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{12}$

337. The remainder when $32^{(32)^{(32)}}$ is divided by 7, is

a) 1 b) 2 c) 3 d) 4

338. If $A = 1000^{1000}$ and $B = (1001)^{999}$, then

a) $A > B$
b) $A = B$
c) $A < B$
d) None of these

339. If the r th term in the expansion of $\left(\frac{x}{3} - \frac{2}{x^2}\right)^{10}$ contains x^4, then r is equal to

a) 3 b) 0 c) -3 d) 5

340. The coefficient of term independent of x in

$$\left[\sqrt{\left(\frac{x}{3}\right)} + \frac{\sqrt{3}}{x^2}\right]^{10}$$ is

a) $\frac{5}{3}$ b) $\frac{4}{5}$ c) 6 d) $\frac{1}{2}$

341. The ratio of the coefficient of x^{15} to the term independent of x in $\left(x^2+\frac{2}{x}\right)^{15}$, is

a) 1/4 b) 1/16 c) 1/32 d) 1/64

342. The sum ${}^{40}C_0 + {}^{40}C_1 + {}^{40}C_2 + \cdots + {}^{40}C_{20}$ is equal to

a) $2^{40}+\dfrac{40!}{(20!)^2}$ b) $2^{39}-\dfrac{1}{2}\times\dfrac{40!}{(20!)^2}$ c) $2^{39}+{}^{40}C_{20}$ d) None of these

343. The coefficient of x^5 in the expansion of $\frac{1+x^2}{1+x}$, $|x|<1$, is

a) -1 b) 2 c) 0 d) -2

344. The coefficient of x^2 term in the binomial expansion of $\left(\frac{1}{3}x^{1/2}+x^{-1/4}\right)^{10}$ is

a) $\dfrac{70}{243}$ b) $\dfrac{60}{423}$ c) $\dfrac{50}{13}$ d) None of these

345. If the expansion of $\left(\dfrac{3\sqrt{x}}{7}-\dfrac{5}{2x\sqrt{x}}\right)^{13n}$ contains a term independent of x in 14th term, then n should be

a) 10 b) 5 c) 6 d) 4

346. The interval in which x must lie so that the greatest term in the expansion of $(1+x)^{2n}$ has the greatest coefficient, is

a) $\left(\dfrac{n-1}{n},\dfrac{n}{n-1}\right)$

b) $\left(\dfrac{n}{n+1},\dfrac{n+1}{n}\right)$

c) $\left(\dfrac{n}{n+2},\dfrac{n+2}{n}\right)$

d) None of these

347. The coefficient of a^3b^4c in the expansion of $(1+a-b+c)^9$is equal to

a) $\dfrac{9!}{3!\,6!}$ b) $\dfrac{9!}{4!\,5!}$ c) $\dfrac{9!}{3!\,5!}$ d) $\dfrac{9!}{3!\,4!}$

348. If $(1+x+x^2)^n=\sum_{r=0}^{2n}a_rx^r$, then $a_1-2a_2+3a_3\ldots-2na_{2n}$ is equal to

a) n b) $-n$ c) 0 d) $2n$

349. In the expansion of $(1+x)^{30}$, the sum of the coefficient of odd powers of x, is

a) 2^{30} b) 2^{31} c) 0 d) 29

350. If the fourth term in the expansion of $\left\{\sqrt{x^{\left(\frac{1}{\log x+1}\right)}}+x^{1/12}\right\}^6$ is equal to 200 and $x>1$, then x is equal to

a) $10^{\sqrt{2}}$ b) 10 c) 10^4 d) None of these

351. The coefficient of x^6 in $\{(1+x)^6+(1+x)^7+\cdots+(1+x)^{15}\}$ is

a) ${}^{16}C_9$ b) ${}^{16}C_5-{}^{6}C_5$ c) ${}^{16}C_6-1$ d) None of these

352. The sum of the series $1+\frac{1}{5}+\frac{1\cdot3}{5\cdot10}+\frac{1\cdot3\cdot5}{5\cdot10\cdot15}+\ldots$ is equal to

a) $\dfrac{1}{\sqrt{5}}$ b) $\dfrac{1}{\sqrt{2}}$ c) $\sqrt{3}$ d) $\sqrt{\dfrac{5}{3}}$

353. If $A=\begin{bmatrix}1&0\\1&1\end{bmatrix}$ and $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, Then which one of the following holds for all $n\geq1$, by the principle of mathematical induction?

a) $A^n=2^{n-1}A+(n-1)I$ b) $A^n=nA+(n-1)I$

c) $A^n=2^{n-1}A-(n-1)I$ d) $A^n=nA-(n-1)I$

354. The greatest term in the expansion of $\sqrt{3}\left(1+\dfrac{1}{\sqrt{3}}\right)^{20}$ is

a) $\frac{26840}{9}$ b) $\frac{24840}{9}$ c) $\frac{25840}{9}$ d) None of these

355. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$, then $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$ is equal to

a) $\frac{n^{n-1}}{(n-1)}!$ b) $\frac{(n+1)^{n-1}}{(n-1)!}$ c) $\frac{(n+1)^n}{n!}$ d) $\frac{(n+1)^{n+1}}{n!}$

356. The expression $\left[x+(x^3-1)^{1/2}\right]^5+\left[x-(x^3-1)^{1/2}\right]^5$ is a polynomial of degree

a) 5 b) 6 c) 7 d) 8

357. The sum of the coefficients of the polynomial $(1+x-3x^2)^{2143}$ is

a) -1 b) 1 c) 0 d) None of these

358. If C_r stands for ${}^nC_r$, the sum of the given series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\cdot[C_0^2-2C_1^2+3C_2^2-\ldots+(-1)^n(n+1)C_n^2]$, where n is an even positive integer, is

a) 0 b) $(-1)^{n/2}(n+1)$

c) $(-1)^n(n+2)$ d) $(-1)^{n/2}(n+2)$

359. The value of $(1.002)^{12}$ upto fourth place of decimal is

a) 1.0242 b) 1.0245 c) 1.0004 d) 1.0254

360. The coefficient of the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is

a) $\frac{1}{3}$ b) $\frac{19}{54}$ c) $\frac{17}{54}$ d) $\frac{1}{4}$

361. If a and d are two complex numbers, then the sum to $(n+1)$ terms of the following series $aC_0-(a+d)C_1+(a+2d)C_2-\ldots+\ldots$ is

a) $\frac{a}{2^n}$ b) na c) 0 d) None of these

362. $2^{3n}-7n-1$ is divisible by

a) 64 b) 36 c) 49 d) 25

363. If n is a positive integer, then $5^{2n+2}-24n-25$ isdivisible by

a) 574 b) 575 c) 675 d) 576

364. If $a_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n}\frac{r}{{}^nC_r}$ equals

a) $(n-1)a_n$ b) na_n c) $\frac{1}{2}na_n$ d) None of these

365. The expression $\frac{1}{\sqrt{4x+1}}\left\{\left(1+\frac{\sqrt{4x+1}}{2}\right)^7-\left(1-\frac{\sqrt{4x+1}}{2}\right)^7\right\}$ is a polynomial in x of degree

a) 7 b) 5 c) 4 d) 3

366. If the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ equal the coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$, then a and b satisfy the relation

a) $ab=1$ b) $\frac{a}{b}=1$ c) $a+b=1$ d) $a-b=1$

367. The middle term in the expansion of $\left(x+\frac{1}{2x}\right)^{2n}$ is

a) $\frac{1\cdot3\cdot5\ldots(2n-3)}{n!}$

b) $\frac{1\cdot3\cdot5\ldots(2n-1)}{n!}$

c) $\frac{1\cdot3\cdot5\ldots(2n+1)}{n!}$

d) None of these

368. The coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+\ldots+(1+x)^{30}$ is

a) ${}^{51}C_5$ b) ${}^9C_5$ c) ${}^{31}C_6-{}^{21}C_6$ d) ${}^{30}C_5+{}^{20}C_5$

369. If $(r+1)^{th}$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of r, is

a) 5 b) 6 c) 4 d) 7

370. If in expansion of $(1+x)^{21}$, the coefficient of x^r and x^{r+1} be equal, then r is equal to

a) 9 b) 10 c) 11 d) 12

371. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$, is

a) $\frac{25840}{9}$ b) $\frac{24840}{9}$ c) $\frac{26840}{9}$ d) None of these

372. The coefficient of x^n in the expansion of $(1+x+x^2+\cdots)^{-n}$, is

a) 1 b) $(-1)^n$ c) n d) $n+1$

373. The coefficient of t^{24} in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is

a) ${}^{12}C_6+2$ b) ${}^{12}C_5$ c) ${}^{12}C_6$ d) ${}^{12}C_7$

374. If the coefficients of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in AP, then the value of r is

a) 6 b) 7 c) 8 d) 9

375. The larger of $99^{50}+100^{50}$ and 101^{50} is

a) $99^{50}+100^{50}$ b) Both are equal c) 101^{50} d) None of these

376. The value of the sum of the series

$3\cdot {}^nC_0-8\cdot {}^nC_1+13\ {}^nC_2-18\cdot {}^nC_3+$ ...upto $(n+1)$ terms is

a) 0 b) 3^n c) 5^n d) None of these

377. The last positive integer n such that $\binom{n-3}{3}+\binom{n-1}{4}>\binom{n}{3}$ is

a) 6 b) 7 c) 8 d) 9

378. If S be the sum of coefficients in the expansion of $(\alpha x+\beta y-\gamma z)^n$, where $(\alpha,\beta,\gamma>0)$, then the value of $\lim_{n\to\infty}\frac{S}{\{S^{1/n}+1\}^n}$ is

a) $e^{\left(\frac{\alpha\beta}{\gamma}\right)}$ b) $e^{\left(\frac{\alpha+\beta-\gamma}{\alpha+\beta-\gamma+1}\right)}$ c) $\frac{\alpha\beta}{\gamma}$ d) 0

379. If the sum of the numerical coefficients in the binomial expansion of $\left(\frac{1}{x}+2x\right)^n$ is equal to 6561, then the term independent of x, is

a) ${}^8C_4$ b) ${}^8C_4\times 2^4$ c) ${}^6C_4\times 2^4$ d) None of these

380. $\sum_{0\le i}\sum_{<j\le 10}{}^{10}C_j\ {}^jC_i$ is equal to

a) $2^{10}-1$ b) 2^{10} c) $3^{10}-1$ d) 3^{10}

381. The coefficient of $x^{2^{m+1}}$ in the expansion of $E=\frac{1}{(1+x)(1+x^2)(1+x^4)(1+x^8)\dots(1+x^{2^m})}$, $|x|<1$ is

a) 3 b) 2 c) 1 d) 0

382. If $n\in N$ such that $(7+4\sqrt{3})^n=I+F$, where $I\in N$ and $0<F<1$. Then, the value of $(I+F)(1-F)$ is

a) 0 b) 1 c) 7^{2n} d) 2^{2n}

383. The largest coefficient in the expansion of $(1+x)^{24}$ is

a) ${}^{24}C_{24}$ b) ${}^{24}C_{13}$ c) ${}^{24}C_{12}$ d) ${}^{24}C_{11}$

384. The coefficient of x^4 in the expansion of $(1+x+x^3+x^4)^{10}$, is

a) ${}^{40}C_4$ b) ${}^{10}C_4$ c) 210 d) 310

385. If the third term in the binomial expansion of $(1+x)^m$ is $-\frac{1}{8}x^2$, then the rational value of m, is

a) 2 b) 1/2 c) 3 d) 4

386. If $C_r={}^nC_r$ and $(C_0+C_1)(C_1+C_2)\dots(C_{n-1}+C_n)=k\frac{(n+1)^n}{n!}$, then the value of k, is

a) $C_0\,C_1\,C_2\dots C_n$ b) $C_1^2C_2^2\dots C_n^2$ c) $C_1+C_2+\cdots+C_n$ d) None of these

387. The coefficient of x^{53} in the expansion

$\sum_{m=0}^{100}{}^{100}C_m\,(x-3)^{100-m}\cdot 2^m$, is

a) ${}^{100}C_{47}$ b) ${}^{100}C_{53}$ c) $-{}^{100}C_{53}$ d) $-{}^{100}C_{100}$

388. If the ratio of the coefficient of third and fourth term in the expansion of $\left(x-\frac{1}{2x}\right)^n$ is 1:2, then the value of n will be

a) 18 b) 16 c) 12 d) -10

389. The coefficient of x^n in the expansion of $(1+x+x^2+\cdots)^{-n}$ is

a) 1 b) $(-1)^n$ c) n d) $n+1$

390. If in the expansion of $(a-2b)^n$, the sum of 4th and 5th term is zero, then the value of $\frac{a}{b}$ is

a) $\frac{n-4}{5}$ b) $\frac{n-3}{2}$ c) $\frac{5}{n-4}$ d) $\frac{5}{2(n-4)}$

391. The coefficient of x^5 in the expansion of $(2-x+3x^2)^6$, is

a) -4692 b) 4692 c) 2346 d) -5052

392. The number of terms whose values depend on x in the expansion of $\left(x^2-2+\frac{1}{x^2}\right)^n$, is

a) $2n+1$ b) $2n$ c) n d) None of these

393. If the coefficients of x^7 and x^8 in $(2+x/3)^n$ are equal, then n is equal to

a) 56 b) 55 c) 45 d) 15

394. The term independent of x in $\left\{\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right\}^{10}$, is

a) $\frac{9}{4}$ b) $\frac{3}{4}$ c) $\frac{5}{4}$ d) $\frac{7}{4}$

395. Coefficient of x^{-4} in $\left(\frac{3}{2}-\frac{3}{x^2}\right)^{10}$ is

a) $\frac{405}{226}$ b) $\frac{504}{289}$ c) $\frac{450}{263}$ d) None of these

396. If the coefficients of $(2r+4)^{\text{th}}$ and $(r-2)^{\text{th}}$ terms in the expansion of $(1+x)^{18}$ are equal, then the value of r, is

a) 5 b) 6 c) 7 d) 9

397. The coefficient of x^6 in the expansion of $(1+x+x^2)^{-3}$, is

a) 6 b) 5 c) 4 d) 3

398. $49^n+16n-1$ is divisible by

a) 3 b) 29 c) 19 d) 64

399. The value of $1^2\cdot C_1+3^2\cdot C_3+5^2\cdot C_5+$...is

a) $n(n-1)^{n-2}+n\cdot 2^{n-1}$ b) $n(n-1)^{n-2}$

c) $n(n-1)\cdot 2^{n-3}$ d) None of these

400. If the coefficient of x^2 and x^3 in the expansion of $(3+ax)^9$ be same, then the value of a is

a) 3/7 b) 7/3 c) 7/9 d) 9/7

401. The coefficient of x in the expansion of $(1+x)(1+2x)(1+3x)\ldots\ldots(1+100x)$ is

a) 5050 b) 10100 c) 5151 d) 4950

402. The positive value of a so that the coefficients of x^5 and x^{15} are equal in the expansion of $\left(x^2+\frac{a}{x^3}\right)^{10}$

a) $\frac{1}{2\sqrt{3}}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) $2\sqrt{3}$

403. In the expansion of $(2-3x^3)^{20}$, if the ratio of 10th term to 11th term is 45/22, then x is equal to

a) $-\frac{2}{3}$ b) $\frac{-3}{2}$ c) $-\sqrt[3]{\frac{2}{3}}$ d) $-\sqrt[3]{\frac{3}{2}}$

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1)	b	2)	b	3)	d	4)	c	157)	b	158)	b	159)	b	160)	c
5)	a	6)	a	7)	a	8)	c	161)	a	162)	b	163)	d	164)	b
9)	b	10)	c	11)	b	12)	a	165)	c	166)	a	167)	c	168)	d
13)	c	14)	c	15)	c	16)	b	169)	d	170)	b	171)	c	172)	b
17)	b	18)	d	19)	c	20)	d	173)	c	174)	a	175)	d	176)	c
21)	c	22)	a	23)	a	24)	c	177)	c	178)	d	179)	a	180)	c
25)	a	26)	d	27)	b	28)	b	181)	a	182)	b	183)	b	184)	b
29)	c	30)	c	31)	c	32)	b	185)	b	186)	d	187)	b	188)	b
33)	b	34)	c	35)	d	36)	d	189)	d	190)	a	191)	a	192)	d
37)	b	38)	a	39)	a	40)	b	193)	b	194)	c	195)	b	196)	c
41)	a	42)	c	43)	b	44)	c	197)	a	198)	b	199)	b	200)	a
45)	d	46)	d	47)	c	48)	b	201)	b	202)	b	203)	d	204)	d
49)	d	50)	b	51)	a	52)	b	205)	a	206)	a	207)	b	208)	a
53)	c	54)	a	55)	a	56)	b	209)	c	210)	b	211)	c	212)	a
57)	a	58)	b	59)	b	60)	b	213)	b	214)	d	215)	d	216)	c
61)	c	62)	c	63)	d	64)	b	217)	a	218)	a	219)	a	220)	b
65)	d	66)	c	67)	d	68)	a	221)	a	222)	c	223)	a	224)	d
69)	d	70)	a	71)	a	72)	c	225)	a	226)	a	227)	a	228)	d
73)	b	74)	a	75)	c	76)	b	229)	c	230)	a	231)	b	232)	a
77)	b	78)	c	79)	b	80)	b	233)	c	234)	d	235)	d	236)	c
81)	c	82)	b	83)	c	84)	b	237)	a	238)	a	239)	a	240)	d
85)	d	86)	d	87)	d	88)	d	241)	a	242)	c	243)	b	244)	b
89)	a	90)	b	91)	c	92)	d	245)	b	246)	a	247)	b	248)	b
93)	b	94)	a	95)	a	96)	b	249)	c	250)	b	251)	c	252)	b
97)	b	98)	b	99)	b	100)	b	253)	b	254)	c	255)	b	256)	b
101)	c	102)	c	103)	b	104)	b	257)	b	258)	c	259)	c	260)	d
105)	c	106)	a	107)	d	108)	c	261)	a	262)	a	263)	a	264)	b
109)	c	110)	d	111)	b	112)	c	265)	b	266)	d	267)	c	268)	d
113)	b	114)	b	115)	c	116)	c	269)	b	270)	b	271)	a	272)	b
117)	b	118)	a	119)	c	120)	b	273)	b	274)	b	275)	b	276)	c
121)	d	122)	b	123)	b	124)	d	277)	a	278)	b	279)	b	280)	b
125)	a	126)	d	127)	d	128)	a	281)	a	282)	c	283)	a	284)	b
129)	d	130)	a	131)	c	132)	d	285)	c	286)	d	287)	c	288)	b
133)	d	134)	c	135)	a	136)	b	289)	b	290)	b	291)	a	292)	b
137)	b	138)	b	139)	c	140)	b	293)	d	294)	b	295)	d	296)	c
141)	c	142)	c	143)	b	144)	b	297)	d	298)	b	299)	b	300)	c
145)	d	146)	b	147)	d	148)	b	301)	b	302)	b	303)	c	304)	b
149)	a	150)	d	151)	a	152)	c	305)	b	306)	d	307)	a	308)	b
153)	a	154)	b	155)	b	156)	b	309)	d	310)	b	311)	d	312)	a

313)	a	314)	a	315)	c	316)	b
317)	d	318)	b	319)	c	320)	c
321)	c	322)	c	323)	c	324)	c
325)	c	326)	a	327)	b	328)	b
329)	d	330)	c	331)	a	332)	a
333)	a	334)	a	335)	a	336)	b
337)	d	338)	a	339)	a	340)	a
341)	c	342)	d	343)	d	344)	a
345)	d	346)	b	347)	d	348)	b
349)	d	350)	b	351)	a	352)	d
353)	d	354)	c	355)	c	356)	c
357)	a	358)	d	359)	a	360)	c

361)	c	362)	c	363)	d	364)	c
365)	d	366)	a	367)	b	368)	c
369)	a	370)	b	371)	a	372)	b
373)	a	374)	d	375)	c	376)	a
377)	d	378)	d	379)	b	380)	c
381)	c	382)	b	383)	c	384)	d
385)	b	386)	a	387)	c	388)	d
389)	b	390)	b	391)	d	392)	b
393)	b	394)	c	395)	d	396)	b
397)	d	398)	d	399)	d	400)	d
401)	a	402)	a	403)	a		

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: HINTS AND SOLUTIONS :

1 **(b)**

We have, $(1+x)^{15} = C_0 + C_1 x + C_2 x^2 + \ldots + C_{15} x^{15}$

$\Rightarrow \dfrac{(1+x)^{15}-1}{x} = C_1 + C_2 x + \ldots C_{15} x^{14}$

On differentiating both sides w.r.t. x, we get

$\dfrac{x \cdot 15(1+x)^{14} - (1+x)^{15} + 1}{x^2} = C_2 + 2C_3 x + \ldots + 14 C_{15} x^{13}$

On putting $x = 1$, we get

$C_2 + 2C_3 + \ldots + 14C_{15} = 15.2^{14} - 2^{15} + 1$

$= 13.2^{14} + 1$

2 **(b)**

It is given that

${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$ are A.P.

$\therefore 2 \cdot {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$2 \cdot \dfrac{(2n)!}{(2n-2)!2!} = \dfrac{(2n)!}{(2n-1)!} + \dfrac{(2n)!}{(2n-3!3!)}$

$\Rightarrow 2\dfrac{(2n)(2n-1)}{2} = 2n + \dfrac{(2n)(2n-1)(2n-2)}{3!}$

$\Rightarrow 6(2n-1) = 6 + (2n-1)(2n-2)$

$\Rightarrow 12n - 6 = 6 + 4n^2 - 6n + 2$

$\Rightarrow 4n^2 - 18n + 14 = 0 \Rightarrow 2n^2 - 9n + 7 = 0$

3 **(d)**

$\dfrac{1+2x}{(1-2x)^2} = (1+2x)(1-2x)^{-2}$

$= (1+2x)\left(1 + \dfrac{2}{1!}(2x) + \dfrac{2 \cdot 3}{2!}(2x)^2 + \ldots + \dfrac{2 \cdot 3 \ldots r}{(r-1)!}(2x)^{r-1} + \dfrac{2 \cdot 3 \cdot 4 \ldots (r+1)(2x)^r}{r!}\right)$

The coefficient of $x^r = 2\frac{r!}{(r-1)!}2^{r-1} + \frac{(r-1)!}{r!}2^r$

$= r2^r + (r+1)2^r = 2^r(2r+1)$

4 **(c)**

Given, $A = {}^{30}C_0 \cdot {}^{30}C_{10} - {}^{30}C_1 \cdot {}^{30}C_{11} + {}^{30}C_2 \cdot {}^{30}C_{12} + \cdots + {}^{30}C_{20} \cdot {}^{30}C_{30}$

$=$ coefficient of x^{20} in $(1+x)^{30}(1-x)^{30}$

$=$ coefficient of x^{20} in $(1+x^2)^{30}$

$=$ coefficient of x^{20} in $\sum_{r=0}^{30} (-1)^r \, {}^{30}C_r (x^2)^r$

$= (-1)^{10} \cdot {}^{30}C_{10}$ {for coefficient of x^{20}, let $r = 10$}

$= {}^{30}C_{10}$

5 **(a)**

We have,

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots + a_{2n} x^{2n} = (1 - x + x^2)^n$

Putting $x = 1$ and -1, we get

$(a_0 + a_2 + a_4 + \cdots) + (a_1 + a_3 + a_5 + \cdots) = 1$ ... (i)

And,

$(a_0 + a_2 + a_4 + \cdots) - (a_1 + a_3 + a_5 \ldots) = 3^n$ ... (ii)

Adding (i) and (ii), we get

$a_0 + a_2 + a_4 + \cdots = \dfrac{3^n + 1}{2}$

6 **(a)**

We know that ,

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n$

On integrating both sides, from 0 to 1, we get

$$\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = \left[C_0 x + \frac{C_1 x^2}{2} + \frac{C_2 x^3}{3} + \cdots + \frac{C_n x^{n+1}}{n+1}\right]_0^1$$

$$\Rightarrow \frac{2^{n+1}-1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1}$$

7 **(a)**

7th term from the beginning in the expansion of $\left(2^{1/3} + \frac{1}{3^{1/3}}\right)^x$ is given by

$T_7 = {}^xC_6(2^{1/3})^{x-6}\left(\frac{1}{3^{1/3}}\right)^6$

7^{th} term from the end in the expansion of $\left(2^{1/3}+\frac{1}{3^{1/3}}\right)^x$ is the $(x+1-7+1)^{th} = (x-5)^{th}$ term from the beginning. Therefore,

$T_{x-5} = {}^xC_{x-6}(2^{1/3})^6\left(\frac{1}{3^{1/3}}\right)^{x-6}$

We have,

$\frac{T_7}{T_{x-5}} = \frac{1}{6}$

$\Rightarrow 6\,T_7 = T_{x-5}$

$\Rightarrow 6\ {}^xC_6\ 2^{\frac{x-6}{3}}3^{-2} = {}^xC_{x-6}\ 2^2 3^{-\left(\frac{x-6}{3}\right)}$

$\Rightarrow 2^{\frac{x-9}{3}} = 3^{-\left(\frac{x-9}{3}\right)}$

$\Rightarrow 6^{\frac{x-9}{3}} = 1 \Rightarrow x-9=0 \Rightarrow x=9$

9 **(b)**

$\because T_{r+1} = {}^{10}C_r(x\sin^{-1}\alpha)^{10-r}\left(\frac{\cos^{-1}\alpha}{x}\right)^r$

$= {}^{10}C_r(\sin^{-1}\alpha)^{10-r}(\cos^{-1}\alpha)^r x^{10-2r}$

$\therefore$ For the term independent of x,

$10-2r=0 \Rightarrow r=5$

$T_{5+1} = {}^{10}C_5(\sin^{-1}\alpha)^5(\cos^{-1}\alpha)^5$

$= {}^{10}C_5(\sin^{-1}\alpha\cos^{-1}\alpha)^5$

Let $f(\alpha) = \sin^{-1}\alpha.\cos^{-1}\alpha$

$= \sin^{-1}\alpha\left(\frac{\pi}{2}-\sin^{-1}\alpha\right)$

Put $\sin^{-1}\alpha = t$

$\therefore f(\alpha) = t\left(\frac{\pi}{2}-t\right)$

$= -\left\{t^2-\frac{\pi}{2}t\right\}$

$= -\left\{\left(t-\frac{\pi}{4}\right)^2-\frac{\pi^2}{16}\right\}$

$= \frac{\pi^2}{16}-\left(t-\frac{\pi}{4}\right)^2$

$\therefore\ f(a) = \frac{\pi^2}{16}-\left(\sin^{-1}\alpha-\frac{\pi}{4}\right)^2$

Maximum value of $f(\alpha)$ is $\frac{\pi^2}{16}$, when $\sin^{-1}\alpha = \frac{\pi}{4}$

Also, $-1\le\alpha\le 1$

$\therefore\ -\frac{\pi}{2}\le\sin^{-1}\alpha\le\frac{\pi}{2}$

Minimum value $f(\alpha) = \frac{\pi^2}{16}-\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)^2 = -\frac{\pi^2}{2}$

$\therefore$ Range is $\left[{}^{10}C_5\left(-\frac{\pi^2}{2}\right)^5, {}^{10}C_5\left(\frac{\pi^2}{16}\right)^5\right]$

ie, $\left[-\frac{{}^{10}C_5\pi^{10}}{2^5}, \frac{{}^{10}C_5\pi^{10}}{2^{20}}\right]$

10 **(c)**

Let $A = \binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}x$

$+\binom{30}{2}\binom{30}{12}-\ldots+\binom{30}{20}\binom{30}{30}$

or $A = {}^{30}C_0.\,{}^{30}C_{10}-{}^{30}C_1.\,{}^{30}C_{11}$

$+{}^{30}C_2.\,{}^{30}C_{12}-\ldots+{}^{30}C_{20}.\,{}^{30}C_{30}$

$=$ coefficient of x^{20} in $(1+x)^{30}(1-x)^{30}$

$=$ coefficient of x^{20} in $(1-x^2)^{30}$

$=$ coefficient of x^{20} in $\sum_{r=0}^{30}(-1)^r\,{}^{30}C_r(x^2)^r$

$= (-1)^{10}\,{}^{30}C_{10}$ (for coefficient of x^{20}, let $r=10$)

$= {}^{30}C_{10}$

11 **(b)**

The rth term of $(a+2n)^n$ is

${}^nC_{r-1}(a)^{n-r+1}(2x)^{r-1}$

$= \frac{n!}{(n-r+1)!\,(r-1)!}a^{n-r+1}(2x)^{r-1}$

$= \frac{n(n-1)\ldots(n-r+2)}{(r-1)!}a^{n-r+1}(2x)^{r-1}$

12 **(a)**

We have, $(1+t^2)^{12}(1+t^{12})(1+t^{24})$

$= (1+{}^{12}C_1t^2$

$+{}^{12}C_2t^4+\ldots+{}^{12}C_6t^{12}+\ldots+{}^{12}C_{12}t^{24}+\ldots)(1$

$+t^{12}+t^{24}+t^{36})$

$\therefore$ Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$

$= {}^{12}C_6+{}^{12}C_{12}+1 = {}^{12}C_6+2$

13 **(c)**

We have,

$\frac{(1+x)^2}{(1-x)^3} = (x^{2v}+2x+1)(1-x)^{-3}$

$\Rightarrow \frac{(1+x)^2}{(1+x)^3} = x^2(1-x)^{-3}+2x(1-x)^{-3}$

$+(1-x)^{-3}$

$\therefore$ Coeff. of x^n in $\frac{(1+x)^2}{(1-x)^3}$

$=$ Coeff. of x^n in $x^2(1-x)^{-3}+$

Coeff. of x^n in $2x(1-x)^{-3}+$ Coeff. of x^n in $(1-x)^{-3}$

$=$ Coeff. of x^{n-2} in $(1-x)^{-3}$

$+2$. Coeff. of x^{n-1} in $(1-x)^{-3}$

$+$ Coeff. of x^n in $(1-x)^{-3}$

$= {}^{n-2+3-1}C_{3-1}+2\cdot{}^{n-1+3-1}C_{3-1}+{}^{n+3-1}C_{3-1}$

$= {}^nC_2+2\cdot{}^{n+1}C_2+{}^{n+2}C_2$

$= \frac{n(n-1)}{2}+2\frac{(n+1)\,n}{2}+(n+2)\frac{(n+1)}{2}$

$= \frac{1}{2}(n^2-n+2^2+2n+n^2+3n+2)$

$= 2n^2+2n+1$

14 **(c)**

On substituting $x = 1$ in $(1 + x - 3x^2)^{3148}$, then sum of coefficient
$= (1 + 1 - 3)^{3148} = (-1)^{3148} = 1$

15 **(c)**

$aC_0 - (a + d)C_1 + (a + 2d)C_2 - (a + 3d)C_3 + \cdots + (-1)^n(a + nd)C_n$

$= \sum_{r=0}^{n}(a + rd)(-1)^r\ {}^nC_r$

$= a\sum_{r=0}^{n}(-1)^r\ {}^nC_r - dn\sum_{r=1}^{n-1}{}^{n-1}C_{r-1}\,(-1)^{r-1}$

$= a \times 0 - dn \times 0 = 0$

16 **(b)**

We have,

$\dfrac{18^3 + 7^3 + 3\cdot18\cdot7\cdot25}{3^6 + 6\cdot243\cdot2 + 15\cdot81\cdot4 + 20\cdot27\cdot8 + 15\cdot9\cdot}$

$= \dfrac{18^3 + 7^3 + 3\cdot18\cdot7\cdot}{{}^6C_0\,3^6 + {}^6C_1\,3^5\cdot2^1 + {}^6C_2\,3^4\cdot2^2 + {}^6C_3\,3^3\,2^3}$

$= \dfrac{(18 + 7)^3}{(3 + 2)^6} = \dfrac{5^6}{5^6} = 1$

17 **(b)**

We have,

$(1 + x^2)^5(1 + x)^4$

$= ({}^5C_0 + {}^5C_1\,x^2 + {}^5C_2\,x^4 + \cdots)$

$\times ({}^4C_0 + {}^4C_1\,x + {}^4C_2\,x^2 + {}^4C_3\,x^3 + {}^4C_4\,x^4)$

$\therefore$ Coefficient of x^5 in $\{(1 + x^2)^5(1 + x)^4\}$

$= {}^5C_2 \times {}^5C_1 + {}^4C_3 \times {}^5C_1 = 60$

18 **(d)**

$({}^nC_0)^2 + ({}^nC_1)^2 + ({}^nC_2)^2 + \cdots + ({}^nC_5)^2$

$= ({}^5C_0)^2 + ({}^5C_1)^2 + ({}^5C_2)^2 + ({}^5C_3)^2 + ({}^5C_4)^2 + ({}^5C_5)^2$

$= 1 + 25 + 100 + 100 + 25 + 1 = 252$

19 **(c)**

Let

$S = (1 + x)^{1000} + 2\,x(1 + x)^{999} + 3\,x^2(1 + x)^{998} + \cdots + 1000\,x^{999}(1 + x) + 1001\,x^{1000}$ ... (i)

$\therefore \dfrac{x}{1 + x}S = x(1 + x)^{999} + 2\,x^2(1 + x)^{998} + \cdots + 1000\,x^{1000} + \dfrac{1001\,x^{1001}}{1 + x}$ ... (ii)

Subtracting (ii) from (i), we get

$\left(1 - \dfrac{x}{x + 1}\right)S = (1 + x)^{1000} + x(1 + x)^{999} + x^2(1 + x)^{998} + \cdots + x^{1000} - \dfrac{1001\,x^{1001}}{1 + x}$

$\Rightarrow S = (1 + x)^{1001} + x\,(1 + x)^{1000} + x^2(1 + x)^{999} + \cdots + x^{1000}(1 + x) - 1001\,x^{1001}$

$\Rightarrow S = (1 + x)^{1001}\dfrac{\left\{1 - \left(\frac{x}{1+x}\right)^{1001}\right\}}{\left\{1 - \frac{x}{1+x}\right\}} - 1001\,x^{1001}$

$\Rightarrow S = (1 + x)^{1002}\left\{1 - \left(\dfrac{x}{1 + x}\right)^{1001}\right\} - 1001\,x^{1001}$

$\Rightarrow S = (1 + x)^{1002} - x^{1001}(1 + x) - 1001\,x^{1001}$

$\therefore$ Coefficient of x^{50} in S is ${}^{1002}C_{50}$

20 **(d)**

$\dfrac{1}{(x - 1)^2(x - 2)} = \dfrac{1}{-2(1 - x)^2\left(1 - \frac{x}{2}\right)}$

$= -\dfrac{1}{2}\left[(1 - x)^{-2}\left(1 - \dfrac{x}{2}\right)^{-1}\right]$

$= -\dfrac{1}{2}\left[(1 + 2x + \cdots)\left(1 + \dfrac{x}{2} + \cdots\right)\right]$

$\therefore$ Coefficient of constant term is $-\frac{1}{2}$.

21 **(c)**

Let $S = 1 + \frac{2\cdot1}{3\cdot2} + \frac{2\cdot5}{3\cdot6}\left(\frac{1}{2}\right)^2 + \frac{2\cdot5\cdot8}{3\cdot6\cdot9}\left(\frac{1}{2}\right)^3 + \ldots$

$= 1 + \dfrac{\frac{2}{3}}{1}\left(\dfrac{1}{2}\right) + \dfrac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{2!}\left(\dfrac{1}{2}\right)^2 + \dfrac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)\left(\frac{8}{3}\right)}{3!}\left(\dfrac{1}{2}\right)^3 + \ldots$

$= \left(1 - \dfrac{1}{2}\right)^{-2/3} = \left(\dfrac{1}{2}\right)^{-2/3} = 2^{2/3} = 4^{1/3}$

$\left[\because (1 - x)^{-n} = 1 + nx + \dfrac{n(n + 1)}{2!}x^2 + \ldots\right]$

22 **(a)**

rth term in the expansion of $\left(3x - \dfrac{2}{x^2}\right)^{15}$ is

$T_r = {}^{15}C_{r-1}(3x)^{15-r+1}\left(\dfrac{-2}{x^2}\right)^{r-1}$

$= {}^{15}C_{r-1}(3)^{15-r+1}(-2)^{r-1}(x)^{15-3r+3}$

For the term independent of x, put

$15 - 3r + 3 = 0 \Rightarrow r = 6$

23 **(a)**

We have, $\sum_{r=0}^{n}\sum_{s=0}^{n}(r + s)(C_r + C_s)$

$= \sum_{r=0}^{n}\sum_{s=0}^{n}(rC_r + rC_s + sC_r + sC_s)$

$= \sum_{r=0}^{n}\left[\sum_{s=0}^{n}rC_r + r\sum_{s=0}^{n}C_s + C_s\sum_{s=0}^{n}s + \sum_{s=0}^{n}sC_s\right]$

$= \sum_{r=0}^{n}\left[(n + 1)r\cdot C_r + r2^n + \dfrac{n(n + 1)}{2}C_r + n\cdot2^{n-1}\right]$

$= (n + 1)n\cdot2^{n-1} + (2^n)\dfrac{n(n + 1)}{2} + \dfrac{n(n + 1)}{2}2^n + n2^{n-1}(n + 1)$

$= n(n + 1)2^n + n(n + 1)2^n$

$= 2n(n + 1)2^n$ ...(i)

Also, $\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)(C_r+C_s)$

$$=\sum_{r=0}^{n}4rC_r+2\sum_{0\le r<s\le n}\sum(r+s)(C_r+C_s)$$

$$\therefore\ 2n(n+1)2^n=4n\cdot 2^{n-1}$$
$$+2\sum_{0\le r<s\le n}\sum(r+s)(C_r+C_s)$$

$$\Rightarrow\sum_{0\le r<s\le n}\sum(r+s)(C_r+C_s)=n^2\cdot 2^n$$

25 **(a)**

We know,

$(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_rx^r+\ldots$ ...(i)

and $\left(1+\frac{1}{x}\right)^n=C_0+C_1\frac{1}{x}+C_2\frac{1}{x^2}+\ldots+C_r\frac{1}{x^r}$

$+C_{r+1}\frac{1}{x^{r+1}}+C_{r+2}\frac{1}{x^{r+2}}\ldots C_n\frac{1}{x^n}$ ...(ii)

On multiplying Eqs. (i) and (ii), equation coefficient of x^r in $\frac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$, we get the value of required expression which is

$${}^{2n}C_{n+r}=\frac{(2n)!}{(n-r)!\,(n+r)!}$$

27 **(b)**

In $(x+a)^{100}+(x-a)^{100}$ n is even

$\therefore$ Total number of terms $=\frac{n}{2}+1=\frac{100}{2}+1=51$

28 **(b)**

Given polynomial is

$(x-1)(x-2)(x-3)\ldots(x-19)(x-20)$

$=x^{20}-(1+2+3+\cdots+19+20)x^{19}$

$+(1\times2+2\times3+\cdots+19\times20)x^{18}$

$-\cdots+(1\times2\times3\times4\times\ldots\times19\times20)$

$\therefore$ Coefficient of $x^{19}=-(1+2+3+\ldots+19+20)$

$=-\left[\frac{20}{2}(1+20)\right]$

$=-10\times21=-210$

29 **(c)**

We know that,

${}^{15}C_0+{}^{15}C_1+{}^{15}C_2+\cdots+{}^{15}C_{15}=2^{15}$

$\Longrightarrow 2({}^{15}C_8+{}^{15}C_9+\cdots+{}^{15}C_{15})2^{15}$ $[\because {}^nC_r={}^nC_{n-r}]$

$\Longrightarrow {}^{15}C_8+{}^{15}C_9+\cdots+{}^{15}C_{15}=2^{14}$

30 **(c)**

The number of terms in the expansion of $(a+b+c)^n$

$$=\frac{(n+1)(n+2)}{2}$$

31 **(c)**

We have,

$$T_{r+1}={}^5C_r(y^2)^{5-r}\left(\frac{c}{y}\right)^r={}^5C_ry^{10-3r}\,c^r$$

This will contain y, if $10-3r=1\Rightarrow r=3$

$\therefore$ Coefficient of $y={}^5C_3\,c^3=10\,c^3$

32 **(b)**

$\because (0.99)^{15}=(1-0.01)^{15}$

$=1-{}^{15}C_1(0.01)+{}^{15}C_2(0.01)^2$

$-{}^{15}C_3(0.01)^3+\ldots$

We want to answer correct upto 4 decimal places and as such, we have left further expansion.

$=1-15(0.01)+\frac{15\cdot14}{1\cdot2}(0.0001)$

$-\frac{15\cdot14\cdot13}{1\cdot2\cdot3}(0.000001)+\ldots$

$=1-0.15+0.0105-0.000455+\ldots$

$=1.0105-0.150455$

$=0.8601$

33 **(b)**

By hypothesis, $2^n=4096=2^{12}\Rightarrow n=12$

Since, n is even, hence greatest coefficient

$$={}^nC_{n/2}={}^{12}C_6=\frac{12\cdot11\cdot10\cdot9\cdot8\cdot7}{1\cdot2\cdot3\cdot4\cdot5\cdot6}=924$$

34 **(c)**

Given that, ${}^nC_{r-1}={}^nC_{r+1}$

$$\Rightarrow\frac{n!}{(n-r+1)(n-r)(n-r-1)!\,(r-1)!}$$
$$=\frac{n!}{(n-r-1)!\,(r+1)(r)(r-1)!}$$

$\Rightarrow r^2+r=n^2-nr+n-nr+r^2-r$

$\Rightarrow n^2-2nr-2r+n=0$

$\Rightarrow (n-2r)(n+1)=0\Rightarrow r=\frac{n}{2}$

35 **(d)**

It is given that

${}^nC_1\,x^{n-1}a^1=240$ ... (i)

${}^nC_2\,x^{n-2}\,a^2=720$ ... (ii)

${}^nC_3\,x^{n-3}\,a^3=1080$ ... (iii)

From (i), (ii) and (iii)

$$\frac{({}^nC_2)^2\,x^{2n-4}a^4}{{}^nC_1\,{}^nC_3\,x^{2n-4}\,a^4}=\frac{720\times720}{240\times1080}$$

$$\Rightarrow\frac{6\,n^2(n-1)^2}{4\,n^2(n-1)(n-2)}=2$$

$$\Rightarrow\frac{3\,(n-1)}{2\,(n-2)}=2$$

$\Rightarrow 3n-3=4n-8\Rightarrow n=5$

36 **(d)**

$$\frac{1}{81^n}(1-10\cdot{}^{2n}C_1+10^2\cdot{}^{2n}C_2-10^3\cdot{}^{2n}C_3+\cdots+10^{2n})$$

$$=\frac{1}{(81)^n}(1-10)^{2n}=1$$

37 **(b)**

We have,

$(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1-1+1)^{n-1}(1+2x) = a_1 + 2a_2x + 3a_3x^2 + ... + 2na_{2n}x^{2n-1}$

On putting $x=-1$ we get

$n(1-1+1)^{n-1}(1-2) = a_1 - 2a_2 + 3a_3 - ... - 2na_{2n}$

$\Longrightarrow a_1 - 2a_2 + 3a_3 - ... - 2na_{2n} = -n$

38 **(a)**

Since $(n+1)^{\text{th}}$ term is the middle term in the expansion of $(1+x)^{2n}$

$\therefore$ Coefficient of the middle term

$= {}^{2n}C_n = \dfrac{(2n)!}{n!\,n!}$

$= \dfrac{(1\cdot 3\cdot 5 \dots (2n-1)(2\cdot 4\cdot 6 \dots (2n-2)(2n))}{n!\,n!}$

$= \dfrac{1\cdot 3\cdot 5 \dots .(2n-1)2^n\, n!}{n!\,n!}$

$= \dfrac{1\cdot 3\cdot 5 \dots (2n-1)2^n}{n!}$

39 **(a)**

We have,

$(1+x)^{10}\left(1+\dfrac{1}{x}\right)^{12} = \dfrac{(1+x)^{22}}{x^{12}}$

$\therefore$ Constant term in $(1+x)^{10}\left(1+\dfrac{1}{x}\right)^{12}$

$=$ Coefficient of x^{12} in $(1+x)^{22}$

$= {}^{22}C_{12} = {}^{22}C_{10}$

40 **(b)**

Given, $a_n = na_{n-1}$

For $n=2$

$a_2 = 2a_1 = 2 \quad (\because a_1 = 1 \text{given})$

$a_3 = 3a_2 = 3(2) = 6$

$a_4 = 4(a_3) = 4(6) = 24$

$a_5 = 5(a_4) = 5(24) = 120$

41 **(a)**

Since, $x(1+x)^n = xC_0 + C_1x^2 + C_2x^3 + \dots + C_nx^{n+1}$

On differentiating w.r.t. x, we get

$(1+x)^n + nx(1+x)^{n-1} = C_0 + 2C_1x + 3C_2x^2 + \dots + (n+1)C_nx^n$

Put $x=1$, we get

$C_0 + 2C_1 + 3C_2 + \dots + (n+1)C_n = 2^n + n2^{n-1}$

$= 2^{n-1}(n+2)$

42 **(c)**

Let T_{r+1} denote the $(r+1)^{th}$ term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$. Then,

$T_{r+1} = {}^nC_r\, x^{3n-5r}(-1)^r$

For this term to contain x^5, we must have

$3n - 5r = 5 \Rightarrow r = \dfrac{3n-5}{5}$

$\therefore$ Coefficient of $x^5 = {}^nC_{\frac{3n-5}{5}}(-1)^{\frac{3n-5}{5}}$

Similarly,

Coefficient of $x^{10} = {}^nC_{\frac{3n-10}{5}}(-1)^{\frac{3n-10}{5}}$

Now,

Coefficient of x^5 + Coefficient of $x^{10} = 0$

$\Rightarrow {}^nC_{\frac{3n-5}{5}}(-1)^{\frac{3n-5}{5}} + {}^nC_{\frac{3n-10}{5}}(-1)^{\frac{3n-10}{5}} = 0$

$\Rightarrow {}^nC_{\frac{3n-5}{5}} = {}^nC_{\frac{3n-10}{5}}$

$\Rightarrow \dfrac{3n-5}{5} + \dfrac{3n-10}{5} = n$

$\Rightarrow 6n - 15 = 5n$

$\Rightarrow n = 15$

43 **(b)**

$(1+x+x^2+x^3)^6 = (1+x)^6(1+x^2)^6$

$= ({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + {}^6C_3x^3 + {}^6C_4x^4 + {}^6C_5x^5 + {}^6C_6x^6) \times ({}^6C_0 + {}^6C_1x^2 + {}^6C_2x^4 + {}^6C_3x^6 + {}^6C_4x^8 + {}^6C_5x^{10} + {}^6C_6x^{12})$

$\therefore$ Coefficient of x^{14} in $(1+x+x^2+x^3)^6$

$= {}^6C_2\cdot{}^6C_6 + {}^6C_4\cdot{}^6C_5 + {}^6C_6\cdot{}^6C_4$

$= 15 + 90 + 15 = 120$

44 **(c)**

The 14th term from the end in the expansion of $(\sqrt{x} - \sqrt{y})^{17}$ is the $(18-14+1)^{\text{th}}$ i.e. 5th term from the beginning and is given by

${}^{17}C_4(\sqrt{x})^{13}(-\sqrt{y})^4 = {}^{17}C_4\, x^{13/2}y^2$

45 **(d)**

Put $x=1$, we get

$(1+2+3+\dots+n)^2 = \sum n^3$

46 **(d)**

We have,

$(1+x+x^2)^n = a_0 + a_1x + a_2x^3 + ... + a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1+x+x^2)^{n-1}(1+2x) = a_1 + 2a_2x + 3a_3x^2 + ... + 2na_{2n}x^{2n-1}$

Now, on putting $x=1$, we get

$n(3)^{n-1}\cdot(3) = a_1 + 2a_2 + 3a_3 + ... + 2na_{2n}$

$\Longrightarrow a_1 + 2a_2 + 3a_3 + ... + 2na_{2n} = n\cdot 3^n$

47 **(c)**

There are total $(n+1)$ factors, let $P(x) = 0$

Let $(x + {}^nC_0)(x + 3\,{}^nC_1)(x + 5\,{}^nC_2) \dots [x + (2n+1)\,{}^nC_n]$

$= a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$

Clearly, $a_n = 1$

and roots of the equation $P(x)=0$ are
$-{}^nC_0, -3\,{}^nC_1, \ldots$
Sum of roots $= -a_{n-1}/a_n$
$= -{}^nC_0 - 3\,{}^nC_1 - 5\,{}^nC_2 \ldots$
$\Rightarrow\ a_{n-1} = (n+1)2^n$

48 **(b)**
${}^{n-2}C_r + 2\cdot{}^{n-2}C_{r-1} + {}^{n-2}C_{r-2}$
$= ({}^{n-2}C_r + {}^{n-2}C_{r-1}) + ({}^{n-2}C_{r-1} + {}^{n-2}C_{r-2})$
$= {}^{n-1}C_r + {}^{n-1}C_{r-1}$ $(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r)$
$= {}^nC_r$

49 **(d)**
$\because \dfrac{1}{(x-1)^2(x-2)} = \dfrac{1}{-2(1-x)^2\left(1-\frac{x}{2}\right)}$
$= -\dfrac{1}{2}\left[(1-x)^{-2}\left(1-\dfrac{x}{2}\right)^{-1}\right]$
$= -\dfrac{1}{2}\left[(1+2x+\ldots)\left(1+\dfrac{x}{2}+\ldots\right)\right]$
$\therefore$ Coefficient of constant term is $-\frac{1}{2}$

50 **(b)**
In the expansion of $\left(x^2+\dfrac{a}{x}\right)^5$, the general term is
$T_{r+1} = {}^5C_r(x^2)^{5-r}\left(\dfrac{a}{x}\right)^r = {}^5C_r\cdot a^r\cdot x^{10-3r}$
For the coefficient of x, put
$10-3r=1 \Rightarrow r=3$
$\therefore$ Coefficient of $x = {}^5C_3a^3 = 10a^3$

52 **(b)**
Coefficient of x^r in the expansion of $(1+x)^{10}$ is
${}^{10}C_r$ and it is maximum for $r=\frac{10}{2}=5$
Hence, Greatest coefficient= ${}^{10}C_5 = \frac{10\,!}{(5\,!)^2}$

53 **(c)**
Given expansion is $\left(\frac{a}{x}+bx\right)^{12}$
$\therefore$ General term, $T_{r+1} = {}^{12}C_r\left(\frac{a}{x}\right)^{12-r}(bx)^r$
$= {}^{12}C_r(a)^{12-r}b^rx^{-12+2r}$
For coefficient of x^{-10}, put
$-12+2r=-10$
$\Rightarrow r=1$
Now, the coefficient of x^{-10} is
${}^{12}C_1(a)^{11}(b)^1 = 12a^{11}b$

55 **(a)**
We have,
$T_{r+1} = {}^{21}C_r\left(\sqrt[3]{\dfrac{a}{\sqrt{b}}}\right)^{21-r}\left(\sqrt{\dfrac{b}{\sqrt[3]{a}}}\right)^r$
$\Rightarrow T_{r+1} = {}^{21}C_r\, a^{7-\frac{r}{2}}\, b^{\frac{2}{3}r-\frac{7}{2}}$
Since the powers of a and b are the same
$\therefore 7-\dfrac{r}{2} = \dfrac{2}{3}r - \dfrac{7}{2} \Rightarrow r=9$

56 **(b)**
$(1-x)^{-4} = 1.x^0 + 4x^1 + \dfrac{4.5}{2}x^2 + \ldots$
$= \left[\dfrac{1.2.3}{6}x^0 + \dfrac{2.3.4}{6}x + \dfrac{3.4.5}{6}x^2\right.$
$\left.+\dfrac{4.5.6}{6}x^3 + \ldots + \dfrac{(r+1)(r+2)(r+3)}{6}x^r + \ldots\right]$
Therefore, $T_{r+1} = \frac{(r+1)(r+2)(r+3)}{6}x^r$

57 **(a)**
We have,
$y = \dfrac{1}{3} + \dfrac{1\cdot 3}{3\cdot 6} + \dfrac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \cdots$
$\Rightarrow y+1 = 1 + \dfrac{1}{3} + \dfrac{1\cdot 3}{3\cdot 6} + \dfrac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \cdots$
Comparing the series on RHS with
$1 + n\,x + \dfrac{n(n-1)}{2\,!}x^2 + \cdots$, we get
$n\,x = \dfrac{1}{3}$ ... (i)
and, $\dfrac{n(n-1)}{2}x^2 = \dfrac{1}{6}$ ... (ii)
Dividing (ii) by square of (i), we get
$\dfrac{n-1}{2\,n} = \dfrac{9}{6} \Rightarrow n = -\dfrac{1}{2}$
$\Rightarrow x = -\dfrac{2}{3}$ [putting $n=-\dfrac{1}{2}$ in (i)]
$\therefore y+1 = (1+x)^n$
$\Rightarrow y+1 = \left(1-\dfrac{2}{3}\right)^{-1/2}$
$\Rightarrow y+1 = \left(\dfrac{1}{3}\right)^{-1/2}$
$\Rightarrow (y+1)^2 = \left(\dfrac{1}{3}\right)^{-1} \Rightarrow y^2+2\,y+1 = 3$
$\Rightarrow y^2+2\,y = 2$

58 **(b)**
$S(k) = 1+3+5\ldots.+(2k-1) = 3+k^2$
Put $k=1$ in both sides, we get
LHS $=1$ and RHS$=3+1=4$
$\Longrightarrow$ LHS $\neq$ RHS
Put $(k+1)$ in both sides in the place of k, we get
LHS $= 1+3+5\ldots.+(2k-1)+(2k+1)$
RHS $= 3+(k+1)^2 = 3+k^2+2k+1$
Let LHS = RHS
Then, $1+3+5\ldots.+(2k-1)+(2k+1)$
$= 3+k^2+2k+1$
$\Longrightarrow 1+3+5+\ldots+(2k-1) = 3+k^2$
If $S(k)$ is true, then $S(k+1)$ is also true.
Hence, $S(k) \Longrightarrow S(k+1)$

59 **(b)**

The general term in the expansion of $(5^{1/6} + 2^{1/8})^{100}$ is given by

$T_{r+1} = {}^{100}C_r(5^{1/6})^{100-r}(2^{1/8})^r$

As 5 and 2 are relatively prime, T_{r+1} will be rational, if

$\frac{100-r}{6}$ and $\frac{r}{8}$ are both integers *ie*, if $100 - r$ is a multiple of 6 and r is a multiple of 8. As $0 \le r \le 100$, multiples of 8 upto 100 and corresponding value of $100 - r$ are

$r = 0, 8, 16, 24, \ldots, 88, 96$

ie, $100 - r = 100, 92, 84, 76, \ldots, 12, 4$

Out of $100 - r$, multiples of 6 are 84, 60, 36, 12

$\therefore$ There are four rational terms

Hence, number of irrational terms is $101 - 4 = 97$

60 **(b)**

We have,

$$T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-r+1}\left(-\frac{2}{x^2}\right)^{r-1}$$

$$\Rightarrow T_r = {}^{10}C_{r-1}\left(\frac{1}{3}\right)^{11-r}(-2)^{r-1}x^{13-3r}$$

For this term to contain x^4, we must have

$13 - 2r = 4 \Rightarrow r = 3$

61 **(c)**

We have, $32^{32} = (2^5)^{32} = 2^{160} = (3-1)^{160}$

$= {}^{160}C_0 3^{160} - {}^{160}C_1 \cdot 3^{159} + \ldots - {}^{160}C_{159} \cdot 3 + {}^{160}C_{160}3^{\circ}$

$= 3m + 1$, where $m \in N$

$32^{(32)^{(32)}} = (32)^{3m+1}$

$= (2^5)^{3m+1} = 2^{15m+5}$

$= 2^{3(5m+1)} \cdot 2^2 = (2^3)^{5m+1} \cdot 2^2$

$= (7+1)^{5m=1} \times 4$

$= \{{}^{5m+1}C_0 7^{5m+1} + {}^{5m+1}C_1 7^{5m} + \ldots + {}^{5m+1}C_{5m+1}7 + {}^{5m+1}C_{5m+1} \cdot 7^0\} \times 4$

$= (7n + 1) \times 4$,

where $n = {}^{5m+1}C_0 7^{5m+1} + \ldots + {}^{5m+1}C_{5m} \cdot 7$

$28n + 4$

Thus, when $32^{(32)^{(32)}}$ is divided by 7, the remainder is 4

62 **(c)**

We have,

$$\left[2^{\log_2\sqrt{9^{x-1}+7}} + \frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right]^7$$

$$= \left[\sqrt{9^{x-1}+7} + \frac{1}{(3^{x-1}+1)^{1/5}}\right]^7$$

$$\therefore T_6 = {}^7C_5\left(\sqrt{9^{x-1}+7}\right)^{7-5}\left[\frac{1}{(3^{x-1}+1)^{1/5}}\right]^5$$

$$= {}^7C_5(9^{x-1}+7)\frac{1}{(3^{x-1}+1)}$$

$$\Rightarrow 84 = {}^7C_5\frac{(9^{x-1}+7)}{(3^{x-1}+1)}$$

$$\Rightarrow 9^{x-1} + 7 = 4(3^{x-1}+1)$$

$$\Rightarrow \frac{3^{2x}}{9} + 7 = 4\left(\frac{3^x}{3} + 1\right)$$

$$\Rightarrow 3^{2x} - 12(3^x) + 27 = 0$$

$$\Rightarrow y^2 - 12y + 27 = 0 \quad (\text{put } y = 3^x)$$

$$\Rightarrow (y-3)(y-9) = 0$$

$$\Rightarrow y = 3, 9 \Rightarrow 3^x = 3, 9 \Rightarrow x = 1, 2$$

63 **(d)**

Here, $P(1) = 2$ and from the equation

$P(k) = k(k+1) + 2$

$\Longrightarrow \quad P(1) = 4$

So, $P(1)$ is not true

Hence, mathematical induction is not applicable.

64 **(b)**

We have,

$(1 + 2x + x^2)^{20} = \{(1+x)^2\}^{20} = (1+x)^{40}$

Clearly, $(1+x)^{40}$ contains 41 terms

Hence, $(1 + 2x + x^2)^{20}$ contains 41 terms

65 **(d)**

The series of binomial coefficient is

$${}^{15}C_8$$

$$\underset{\text{decreasing value}}{\xleftarrow{{}^{15}C_0, {}^{15}C_1, {}^{15}C_2, \ldots, {}^{15}C_7}} \quad \downarrow \quad \underset{\text{decreasing value}}{\xrightarrow{{}^{15}C_9, \ldots, {}^{15}C_9, {}^{15}C_{15}}}$$

From the above discussion, we can say that decreasing series is ${}^{15}C_7$, ${}^{15}C_6$, ${}^{15}C_5$.

66 **(c)**

For $n = 1$, $10^n + 3 \cdot 4^{n+2} + 5$

$= 10 + 3 \cdot 4^3 + 5 = 207$ This is divisible by 9.

$\therefore$ By induction, the result is divisible by 9.

67 **(d)**

$$\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + \cdots + {}^8C_8 \cdot 6^7$$

$$= \frac{1}{6}[{}^8C_0 - 6\,{}^8C_1 + 6^2\,{}^8C_2 - 6^3\,{}^8C_3 + \cdots + 6^8\,{}^8C_8]$$

$$= \frac{1}{6}[(1-6)^8] = \frac{5^8}{6}$$

68 **(a)**

In the expansion of $(1+x)^n$,it is given that

${}^nC_1$, ${}^nC_2$, ${}^nC_3$ are in AP

$\Rightarrow 2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$$\Rightarrow 2 \cdot \frac{n(n-1)}{1 \cdot 2} = \frac{n}{1} + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}$$

$$\Rightarrow 6(n-1) = 6 + (n-2)(n-1)$$

$$\Rightarrow 6n - 6 = 6 + n^2 - 3n + 2$$

$\Rightarrow n^2-9n+14=0$

$\Rightarrow (n-2)(n-7)=0$

$\Rightarrow n=2,7$

But $n=2$ is not acceptable because, when $n=2$, there are only three terms in the expansion of $(1+x)^2$

$\therefore n=7$

70 **(a)**

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$ ...(i)

On differentiating both sides w. r. t. x, we get

$n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2x + \ldots + n\,{}^nC_nx^{n-1}$ ...(ii)

On putting $x=1$ in Eq. (ii), we get

$n(2)^{n-1} = {}^nC_1 + 2\,{}^nC_2 + \ldots + n\,{}^nC_n$ ...(iii)

On putting $x=-1$ in Eq. (ii) we get

$0 = {}^nC_1 - 2\,{}^nC_2 + 3\,{}^nC_3 - \ldots (-1)^{n-1}\cdot {}^nC_n$ ...(iv)

On adding Eqs. (iii) and (iv), we get

$n2^{n-1} = 2({}^nC_1 + 3\,{}^nC_3 + ..)$

$\Longrightarrow {}^nC_1 + 3\,{}^nC_3 + 5\,{}^nC_5 + \ldots = \frac{n}{2}\cdot 2^{n-1} = n2^{n-2}$

71 **(a)**

Given expression is $\left(x + x^{\log 10^x}\right)^5$

$\therefore T_3 = {}^5C_2\cdot x^3\left(x^{\log 10^x}\right)^2 = 10^6$ (given)

Put $x=10$, then $10^4.10^2 = 10^6$ is satisfied.

Hence, $x=10$.

72 **(c)**

Given, ${}^nC_0 - \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 - \ldots + (-1)^n \frac{{}^nC_n}{n+1}$

At $n=1$, ${}^1C_0 - \frac{1}{2}\,{}^1C_1 = 1 - \frac{1}{2} = \frac{1}{2}$

At $n=2$, ${}^2C_0 - \frac{1}{2}\,{}^2C_1 + \frac{1}{3}\,{}^2C_2 = 1 - 1 + \frac{1}{3} = \frac{1}{3}$

Which is satisfied only in option (c)

73 **(b)**

$8^{2n} - (62)^{2n+1} = (1+63)^n - (63-1)^{2n+1}$

$= (1+63)^n + (1-63)^{2n+1}$

$= (1 + {}^nC_163 + {}^nC_2(63)^2 + \cdots + (63)^n)$

$+ (1 - {}^{(2n+1)}C_163 + {}^{(2n+1)}C_2(63)^2 + \cdots + (-1)(63)^{(2n+1)})$

$= 2 + 63[{}^nC_1 + {}^nC_2(63) + \ldots + (63)^{n-1} - {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2(63) - \ldots - (63)^{(2n)}]$

$\therefore$ Remainder is 2.

74 **(a)**

We have,

$T_{r+1} = {}^{20}C_r \times 4^{\frac{20-r}{3}} \times 6^{-\frac{r}{4}}$

$\Rightarrow T_{r+1} = {}^{20}C_r 2^{\frac{160-11r}{12}} 3^{-\frac{r}{4}}, r = 0,1,2,\ldots,20$

This term will be rational if $\frac{160-11r}{12}$ and $\frac{r}{4}$ are rational numbers

Now, $\frac{r}{4}$ is rational if $r = 0,4,8,12,16,20$

Clearly, $\frac{160-11r}{12}$ is rational for $= 8,16$ and 20

Hence, there are only 3 rational terms

75 **(c)**

We have,

$\left(x^2 + 1 + \frac{1}{x^2}\right)^n = \frac{1}{x^{2n}}(1 + x^2 + x^4)^n$

$= \frac{1}{t^n}(1 + t + t^2)^n$, where $t = x^2$

Clearly, $(1+t+t^2)^n$ is a polynomial of degree $2n$

Hence, there are $(2n+1)$ terms

76 **(b)**

$(19)^{2005} + (11)^{2005} - (9)^{2005}$

$= (10+9)^{2005} + (10+1)^{2005} - (9)^{2005}$

$= (9^{2005} + {}^{2005}C_1(9)^{2004} \times 10 + ...) + ({}^{2005}C_0 + {}^{2005}C_1 10 + \ldots) - (9)^{2005}$

$= ({}^{2005}C_1 9^{2005} \times 10 +$ multipale of $10) + (1 +$ multipal of $10)$

$\therefore$ Unit digit=1

77 **(b)**

In the expansion of $(x+2y)^6$,

$\left(\frac{6}{2}+1\right)$th term is the middle term.

$\therefore \quad T_4 = T_{3+1} = {}^6C_3x^{6-3}(2y)^3$

$= \quad 8({}^6C_3)(xy)^3$

$\therefore$ Coefficient of middle term

$= 8({}^6C_3)$

78 **(c)**

General terms, $T_{r+1} = (1)^r\ {}^{15}C_r\ (x^4)^{15-r}\cdot\left(\frac{1}{x^3}\right)^r$

$= (-1)^r\,{}^{15}C_r\cdot x^{60-7r}$

For the coefficient of x^{-17}, put $60-7r=-17$

$\Longrightarrow \quad 60+17 = 7r \Longrightarrow r = 11$

Now, coefficient of $x^{-17} = (-1)^{11}\,{}^{15}C_{11} = -{}^{15}C_{11}$

79 **(b)**

$$\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{2\left(1-\frac{x}{4}\right)^{1/2}}$$

$$= \frac{\left[\left[1 + \frac{1}{2}(-3x) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(-3x)^2 + \ldots\right] + \left[1 + \frac{5}{3}(-x) + \frac{5}{3}\cdot\frac{2}{3}\cdot\frac{1}{2}(-x)^2 + \ldots\right]\right]}{2\left[1 + \frac{1}{2}\left(-\frac{x}{4}\right) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}\left(-\frac{x}{4}\right)^2 + \ldots\right]}$$

$$= \frac{2\left[1 - \frac{19}{12}x - \frac{41}{144}x^2 - \ldots\right]}{2\left[1 - \frac{x}{8} - \frac{1}{128}x^2 - \ldots\right]}$$

$$= \left[1 - \frac{19}{12}x - \frac{41}{144}x^2 - \ldots\right]\left[1 - \frac{x}{8} - \frac{1}{128}x^2 \ldots\right]^{-1}$$

$$= 1 - \frac{35}{24}x + \ldots$$

On neglecting higher powers of x, we get

$$a + bx = 1 - \frac{35}{24}x$$

$$\Rightarrow a = 1, b = -\frac{35}{24}$$

80 **(b)**

$${}^{18}C_{15} + 2({}^{18}C_{16}) + {}^{17}C_{16} + 1 = {}^{n}C_3$$

$$\Rightarrow {}^{18}C_{15} + {}^{18}C_{16} + {}^{18}C_{16} + {}^{17}C_{16} + {}^{17}C_{17} = {}^{n}C_3$$

$$\Rightarrow {}^{19}C_{16} + {}^{18}C_{16} + {}^{18}C_{17} = {}^{n}C_3$$

$$\Rightarrow {}^{19}C_{16} + {}^{19}C_{17} = {}^{n}C_3$$

$$\Rightarrow {}^{20}C_{17} = {}^{n}C_3 \Rightarrow {}^{20}C_3 = {}^{n}C_3 \Rightarrow n = 20$$

81 **(c)**

We have, $49^n + 16n - 1 = (1+48)^n + 16n - 1$

$$= 1 + {}^{n}C_1(48) + {}^{n}C_2(48)^2 + \ldots + {}^{n}C_n(48)^n + 16n - 1$$

$$= (48n + 16n) + {}^{n}C_2(48)^2 + {}^{n}C_3(48)^3 + \ldots + {}^{n}C_n(48)^n$$

$$= 64n + 8^2[{}^{n}C_2 \cdot 6^2 + {}^{n}C_3 \cdot 6^3 \cdot 8 + {}^{n}C_4 \cdot 6^4 \cdot 8^2 + \ldots + {}^{n}C_n \cdot 6^n \cdot 8^{n-2}]$$

Hence, $49^n + 16n - 1$ is divisible by 64

82 **(b)**

We have, $(1+x)^{50} = \sum_{r=0}^{50} {}^{50}C_r x^r$. (The sum of coefficients of odd powers of x)

$$= {}^{50}C_1 + {}^{50}C_3 + \ldots + {}^{50}C_{49}$$

$$= 2^{50-1} = 2^{49}$$

84 **(b)**

Given, $\alpha = \frac{5}{2!\,3} + \frac{5 \cdot 7}{3!\,3^2} + \frac{5 \cdot 7 \cdot 9}{4!\,3^2} + \cdots$ ... (i)

On comparing

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \quad \text{... (ii)}$$

With respect to factorial, we get

$$n(n-1)x^2 = \frac{5}{3} \quad \text{.... (iii)}$$

$$n(n-1)(n-2)x^3 = \frac{5 \cdot 7}{3^2} \quad \text{... (iv)}$$

and $n(n-1)(n-2)(n-3)x^4 = \frac{5 \cdot 7 \cdot 9}{3^3}$ ..(v)

on dividing Eq. (iv) by (iii) and Eq. (v) by Eq. (iv), we get

$$(n-2)x = \frac{7}{3} \quad \text{... (vi)}$$

and $(n-3)x = 3$ ...(vii)

Again, dividing Eq. (vi) by Eq. (vii), we get

$$\frac{n-2}{n-3} = \frac{7}{9}$$

$$\Rightarrow 9n - 18 = 7n - 21$$

$$\Rightarrow 2n = -3 \Rightarrow n = -\frac{3}{2}$$

On putting the value of n in Eq. (vi), we get

$$\left(-\frac{3}{2} - 2\right)x = \frac{7}{3} \Rightarrow x = -\frac{2}{3}$$

$\therefore$ From Eq. (ii),

$$\left(1 - \frac{2}{3}\right)^{-3/2} = 1 + 1 + \frac{5}{2!\,3} + \frac{5 \cdot 7}{3!\,3^2} + \cdots$$

$$\Rightarrow 3^{3/2} - 2 = \frac{5}{2!\,3} + \frac{5 \cdot 7}{3!\,3^2} + \ldots$$

$$\Rightarrow \alpha = 3^{3/2} - 2 \quad \text{[from Eq. (i)]}$$

Now, $\alpha^2 + 4\alpha = (3^{3/2} - 2)^2 + 4(3^{3/2} - 2)$

$$= 27 + 4 - 4 \cdot 3^{3/2} + 4 \cdot 3^{3/2} - 8$$

$$= 23$$

85 **(d)**

$$\frac{1+2x}{(1-2x)^2} = (1+2x)(1-2x)^{-2}$$

$$= (1+2x)\left(1 + \frac{2}{1!}(2x) + \frac{2 \cdot 3}{2!}(2x)^2 + \cdots + \frac{2 \cdot 3 \ldots r}{(r-1)!}(2x)^r + \frac{2 \cdot 3 \cdot 4 \ldots (r+1)(2x)^r}{r!}\right)$$

The coefficient of x^r

$$= 2\frac{r!}{(r-1)!}2^{r-1} + \frac{(r+1)!}{r!}2^r$$

$$= r2^r + (r+1)r^2$$

$$= 2^r(2r+1)$$

86 **(d)**

We have,

$$\{(1+x)(1+y)(x+y)\}^n = (1+x)^n(1+y)^n(x+y)^n$$

$\therefore$ Coefficient of $x^n y^n$ in $\{(1+x)(1+y)(x+y)\}^n$

$$= \sum_{r=0}^{n}({}^{n}C_r)^3$$

87 **(d)**

We have,

$$(1+x+x^2)^n = C_0 + C_1x + C_2x^2 + \cdots + C_{2n}x^{2n}$$

Replacing x by $-\frac{1}{x}$, we get

$$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n = C_0 - C_1\frac{1}{x} + C_2\frac{1}{x^2} + \cdots + C_{2n}\frac{1}{x^{2n}}$$

Now,

$$C_0C_1 - C_1C_2 + C_2C_3 - \cdots$$

$$= \text{Coeff. of } x \text{ in } \{C_0 + C_1x + C_2x^2 + \cdots\}\left\{C_0 - C_1\frac{1}{x} + C_2\frac{1}{x^2} - \cdots\right\}$$

$$= \text{Coeff. of } x \text{ in } (1+x+x^2)^n\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n$$

$$= \text{Coeff. of } x^{2n+1} \text{ in } (1+x+x^2)^n(x^2 - x + 1)^n$$

$$= \text{Coeff. of } x^{2n+1} \text{ in } [(1+x^2)^2 - x^2]^2$$

$= \text{Coeff. of } x^{2n+1} \text{ in } [1+x^2+x^4]^n = 0$

88 **(d)**

$\because a, b, c$ are in AP

$\Rightarrow 2b = a + c$

$\Rightarrow a - 2b + c = 0$

On putting $x = 1$, we get

Required sum $= (1 + (a - 2b + c)^2)^{1973} =$
$(1+0)^{1973} = 1$

89 **(a)**

We have , $T_2 = 14a^{5/2}$

$\Longrightarrow {}^nC_1(a^{1/13})^{n-1}(a^{3/2})^1 = 14a^{5/2}$

$\Longrightarrow na^{\frac{n-1}{13}+\frac{3}{2}} = 14a^{5/2}$

$\Longrightarrow \quad n = 14$

$\therefore \frac{{}^nC_3}{{}^nC_2} = \frac{{}^{14}C_3}{{}^{14}C_2} = 4$

90 **(b)**

For $n > 1$, we have

$$(1+x)^n = {}^nC_0 + {}^nC_1\,x + {}^nC_2\,x^2 + {}^nC_3\,x^3 + \cdots + {}^nC_n\,x^n$$

$$\Rightarrow (1+x)^n = 1 + nx + ({}^nC_2\,x^2 + {}^nC_3\,x^3 + \cdots + {}^nC_n\,x^n)$$

$$\Rightarrow (1+x)^n - 1 - nx = x^2({}^nC_2 + {}^nC_3\,x + {}^nC_4\,x^2 + \cdots + {}^nC_n\,x^{n-2})$$

Clearly, RHS is divisible by x^2 and x. So, LHS is also divisible by x as well as x^2

91 **(c)**

Let T_{r+1} be the $(r+1)^{th}$ terms in the expansion of $\left(\frac{x^2}{a} - \frac{a}{x}\right)^{12}$. Then,

$$T_{r+1} = {}^{12}C_r\left(\frac{x^2}{a}\right)^{12-r}\left(-\frac{a}{x}\right)^r = {}^{12}C_r\,x^{24-3r}(-1)^r\,a^{2r-12}$$

For the coefficient of $x^6\,y^{-2}$, we must have

$24 - 3r = 6$ and $2r - 12 = -2$

These two equations are inconsistent

Hence, there is no term containing $x^6\,a^{-2}$

So, its coefficient is 0

92 **(d)**

$\because I + f + f' = (5+2\sqrt{6})^n + (5-2\sqrt{6})^n$
$= 2k$(even integer)

$\therefore f + f' = 1$

Now, $(I+f)f' = (5+2\sqrt{6})^n(5-2\sqrt{6})^n =$
$(1)^n = 1$

$\Rightarrow (I+f)(1-f) = 1$

or $I = \frac{1}{(1-f)} - f$

93 **(b)**

Given equation can be rewritten as

$$E = a[{}^nC_0 - {}^nC_1 + {}^nC_2 - \ldots + (-1)^n\,{}^nC_n] + [{}^nC_1 - (2)({}^nC_2) + (3)({}^nC_3) - \ldots + (-1)^n(n)\,({}^nC_n)]$$

$\Rightarrow E = 0 + 0 = 0$ (by properties)

94 **(a)**

Coefficient of x^{r-1} in

$(1+x)^n + (1+x)^{n+1} + \ldots + (1+x)^{n+k}$

$= {}^nC_{r-1} + {}^{n+1}C_{r-1} + \ldots + {}^{n+k}C_{r-1}$

$= {}^nC_r + {}^nC_{r-1} + {}^{n+1}C_{r-1} + \ldots + {}^{n+k}C_{r-1} - {}^nC_r$

$= {}^{n+k+1}C_r - {}^nC_r$

Now, $\sum_{r=0}^{n+k+1}(-1)^r a_r = \sum_{r=0}^{n+k+1}(-1)^r\ {}^{n+k-1}C_r -$
$\sum_{r=0}^{n+k+1}(-1)^r\ {}^nC_r = 0$

95 **(a)**

We have, $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$

If x is replace by $-\left(1-\frac{1}{x}\right)$ and n is $-n$, then expression

becomes $\left[1 - \left(1 - \frac{1}{x}\right)\right]^{-n}$

$$= 1 + (-n)\left[-\left(1-\frac{1}{x}\right)\right] + \frac{(-n)(-n-1)}{2!}\left[-\left(1-\frac{1}{x}\right)\right]^2 + \ldots$$

$$\Rightarrow x^n = 1 + n\left(1-\frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2 + \ldots$$

96 **(b)**

Given expansion is$(x+a)^n$

On replacing a by ai and $-ai$ respectively, we get

$(x+ai)^n = (T_0 - T_2 + T_4 - \ldots) + i(T_1 - T_3 + T_5 - \ldots)$ ...(i)

and $(x-ai)^n = (T_0 - T_2 + T_4 - \ldots) + i(T_1 - T_3 + T_5 - \ldots)$ ...(ii)

On multiplying Eqs. (ii) and (i), we get required result

$$(x^2+a^2)^n = (T_0 - T_2 + T_4 - \ldots)^2 + (T_1 - T_3 + T_5 - \ldots)^2$$

97 **(b)**

Given coefficient of $(2x+1)$th term=coefficient of $(r+2)$th term

$\Rightarrow {}^{43}C_{2r} = {}^{43}C_{r+1}$

$\Rightarrow 2r + (r+1) = 43$ or $2r = r + 1$

$\Rightarrow r = 14$ or $r = 1$

98 **(b)**

We have,

$(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ ...(i)

and $\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} +$
$C_2\left(\frac{1}{x}\right)^2 + \ldots + C_n\left(\frac{1}{x}\right)^n$ ...(ii)

On multiplying Eqs. (i) and (ii) and taking the coefficient of constant terms in right hand side

$= C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2$

In right hand side $(1+x)^n\left(1+\frac{1}{x}\right)^n$ or in $\frac{1}{x^n}(1+x)^{2n}$ or term containing x^n in $(1+x)^{2n}$.

Clearly the coefficient of x^n in $(1+x)^{2n}$ is equal to ${}^{2n}C_n = \frac{(2n)!}{n!n!}$

99 **(b)**

We have,

$$\frac{C_k}{C_{k-1}} = \frac{{}^nC_k}{{}^nC_{k-1}} = \frac{n-k+1}{k}$$

$$\therefore \sum_{k=1}^{n} k^3\left(\frac{C_k}{C_{k-1}}\right)^2$$

$$= \sum_{k=1}^{n} k^3 \frac{(n-k+1)^2}{k^2} = \sum_{k=1}^{n} k(n-k+1)^2$$

$$= (n+1)^2\left(\sum_{k=1}^{n} k\right) - 2(n+1)\left(\sum_{k=1}^{n} k^2\right) + \left(\sum_{k=1}^{n} k^3\right)$$

$$= (n+1)^2\frac{n(n+1)}{2} - \frac{2(n+1)n(n+1)(2n+1)}{6} + \left\{\frac{n(n+1)}{2}\right\}^2$$

$$= \frac{n(n+1)^2}{12}\{6(n+1) - 4(2n+1) + 3n\}$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

100 **(b)**

Let

$$S = 1\times2\times3\times4 + 2\times3\times4\times5 + 3\times4\times5\times6 + \cdots + n(n+1)(n+2)(n+3)$$

$$\Rightarrow S = \sum_{r=1}^{n} r(r+1)(r+2)(r+3)$$

$$\Rightarrow S = \sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!}$$

$$\Rightarrow S = 4!\sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!\,4!}$$

$$\Rightarrow S = 4!\sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!\,4!}$$

$$\Rightarrow S = 4!\sum_{r=1}^{n} {}^{r+3}C_4$$

$$\Rightarrow S = 4!\sum_{r=12}^{n} \text{Coefficient of } x^4 \text{in } (1+x)^{r+3}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^4 \text{ in } \sum_{r=1}^{n}(1+x)^{r+3}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^4 \text{ in } (1+x)^4\left\{\frac{(1+x)^n - 1}{(1+x)-1}\right\}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^5 \text{ in } \{(1+x)^{n+4} - (1+x)^4\}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^5 \text{ in } (1+x)^{n+4}$$

$$\Rightarrow S = 4! \times {}^{n+4}C_5$$

$$= \frac{1}{5}n(n+1)(n+2)(n+3)(n+4)$$

101 **(c)**

We have,

$$(x+y+z)^{18} = \sum_{r+s+t=18} \frac{18!}{r!\,s!\,t!} x^r\, y^s\, z^t$$

$$\therefore \text{Coefficient of } x^8y^6z^4 = \frac{18!}{8!\,6!\,4!} = \frac{18!}{10!\,8!} \times \frac{10!}{6!\,4!}$$

$$= {}^{18}C_{10} \times {}^{10}C_6$$

$$\text{Also, Coefficient of } x^8y^6z^4 = \frac{18!}{8!\ 6!\ 4!} = \frac{18!}{4!\ 14!} \times \frac{14!}{8!\ 6!}$$

$$= {}^{18}C_{14} \times {}^{14}C_8 = {}^{18}C_4 \times {}^{14}C_6$$

Again,

$$\text{Coefficient of } x^8y^6z^4 = \frac{18!}{8!\,6!\,4!} = \frac{18!}{12!\,6!} \times \frac{12!}{8!\,4!}$$

$$= {}^{18}C_6 \times {}^{12}C_8$$

102 **(c)**

We have,

$$1 + x + x^2 + x^3 = (1+x)(1+x^2)$$

$$\therefore (1+x+x^2+x^3)^{11} = (1+x)^{11}(1+x^2)^{11}$$

$$= ({}^{11}C_0 + {}^{11}C_1\,x + {}^{11}C_2\,x^2 + {}^{11}C_3\,x^3 + {}^{11}C_4\,x^4 + \cdots) \times ({}^{11}C_0 + {}^{11}C_1\,x^2 + {}^{11}C_2\,x^4 + \cdots)$$

$$\Rightarrow \text{Coefficient of } x^4 \text{ in } (1+x+x^2+x^3)^{11}$$

$$= \text{Coefficient of } x^4 \text{ in } \{({}^{11}C_0 + {}^{11}C_1\,x + {}^{11}C_2\,x^2 + \cdots)({}^{11}C_0 + {}^{11}C_1\,x^2 + {}^{11}C_2\,x^4 + \cdots)\}$$

$$= {}^{11}C_0 \times {}^{11}C_2 + {}^{11}C_2 \times {}^{11}C_1 + {}^{11}C_4 \times {}^{11}C_0 = 990$$

103 **(b)**

We have,

a = Sum of the coefficients in the expansion of $(1 - 3x + 10\,x^2)^n$

$\Rightarrow a = (1 - 3 + 10)n = 8^n = 2^{3n}$

b = Sum of the coefficients in the expansion of $(1 + x^2)^n$

$\Rightarrow b = (1+1)^n = 2^n$

Clearly, $a = b^3$

104 **(b)**

Let $P(n): 10^{n-2} \geq 81n$

For $n = 4, 10^2 \ngeq 81 \times 4$

For $n = 5, 10^3 \geq 81 \times 5$

Hence, by mathematical induction for $n \geq 5$, the proposition is true.

105 **(c)**

Given that, $T_1 = {}^nC_0 = 1$ ...(i)

$T_2 = {}^nC_1 ax = 6x$

$\Rightarrow \frac{n!}{(n-1!)} a = 6 \Rightarrow na = 6$ ...(ii)

and $T_3 = {}^nC_2(ax)^2 = 6x^2$

$\Rightarrow \frac{n(n-1)}{2} a^2 = 16$ ...(iii)

Only option (c) is satisfying Eqs. (ii) and (iii)

106 **(a)**

It is given that

$$(a + b\,x)^{-2} = \frac{1}{4} - 3\,x$$

$$\Rightarrow a^{-2}\left(1 + \frac{b}{a}x\right)^{-2} = \frac{1}{4} - 3\,x$$

$$\Rightarrow a^{-2}\left(1 - \frac{2}{a}b\,x\right) = \frac{1}{4} - 3\,x \quad \left[\begin{array}{l}\text{Neglecting } x^2 \text{ and} \\ \text{higher powers of } x\end{array}\right]$$

$$\Rightarrow a^{-2} = \frac{1}{4}, \frac{-2\,b}{a^3} = -3$$

Now, $a^{-2} = \frac{1}{4} \Rightarrow a^2 = 4 \Rightarrow a = 2 \quad [\because a > 0]$

Putting $a = 2$ in $-\frac{2\,b}{a^3} = -3$, we get $-\frac{2\,b}{8} = -3 \Rightarrow b = 12$

107 **(d)**

We have,

$$T_{r+1} = {}^{15}C_r (x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r = {}^{15}C_r\, x^{60-7r}\,(-1)^r$$

If x^{39} occurs in T_{r+1}, then

$60 - 7r = 39 \Rightarrow r = 3$

$\therefore$ Coefficient of $x^{39} = {}^{15}C_3(-1)^3 = -455$

108 **(c)**

$(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + a_3y^3 + \ldots$

On differentiating w. r. t. y, we get

$-m(1-y)^{m-1}(1+y)^n + (1-y)^m\, n(1+y)^{n-1}$
$= a_1 + 2a_2y + 3a_3y^2 + \ldots$ ...(i)

On putting $y = 0$ in Eq. (i), we get

$-m + n = a_1 = 10 \quad [\because a_1 = 10 \text{ given}]$ ...(ii)

Again on differentiating Eq. (i) w. r. t. y, we get

$-m[-(m-1)(1-y)^{m-2}(1+y)^n + (1+y)^{m-1}n(1+y)^{n-1}]$
$+n[-m(1-y)^{m-1}(1+y)^{n-1} + (1-y)^m(n-1)(1+y)^{n-2}]$
$= 2a_2 + 6a_3y + \ldots$ ...(iii)

On putting $y = 0$ in Eq. (iii), we get

$-m[-(m-1)+n] + n[-m+(n-1)] = 2a_2 = 20$

$\Rightarrow m(m-1) - mn - mn + n(n-1) = 20$

$\Rightarrow m^2 + n^2 - m - n - 2mn = 20$

$\Rightarrow (m-n)^2 - (m+n) = 20$

$\Rightarrow 100 - (m+n) = 20$ [using Eq. (iii)]

$\Rightarrow m + n = 80$ ...(iv)

On solving Eqs. (ii) and (iv), we get

$m = 35$ and $n = 45$

109 **(c)**

Let a_1, a_2, a_3, a_4 be respectively the coefficients of $(r+1)$th, $(r+2)$th, $(r+3)$th and $(r+4)$th terms in the expansion of $(1+x)^n$. Then,

$a_1 = {}^nC_r, a_2 = {}^nC_{r+1}, a_3 = {}^nC_{r+2}, a_4 = {}^nC_{r+3}$

Now, $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} + \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}}$

$= \frac{{}^nC_r}{{}^{n+1}C_{r+1}} + \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}} \quad (\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$

$= \frac{{}^nC_r}{\frac{n+1}{r+1}\,{}^nC_r} + \frac{{}^nC_{r+2}}{\frac{n+1}{r+3}\,{}^nC_{r+2}} \quad \left(\because {}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-2}\right)$

$= \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2(r+2)}{n+1}$

$= 2\frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}} = 2\frac{{}^nC_{r+1}}{{}^nC_{r+1} + {}^nC_{r+2}}$

$= \frac{2a_2}{a_2 + a_3}$

110 **(d)**

$(a^2 - 6a + 11)^{10} = 1024$

$\Rightarrow (a^2 - 6a + 11)^{10} = 2^{10}$

$\Rightarrow a^2 - 6a + 11 = 2$

$\Rightarrow a^2 - 6a + 9 = 0$

$\Rightarrow (a-3)^2 = 0$

$\Rightarrow a = 3$

111 **(b)**

The general termof $\left(x + \frac{2}{x^2}\right)^n$ is

$$T_{R+1} = {}^nC_R(x)^{n-R}\left(\frac{2}{x^2}\right)^R = {}^nC_R x^{n-3R}2^R$$

For x^{2r} occurs, it means

$n - 3R = 2r$

$\Rightarrow n - 2r = 3R$

Hence, $n - 2r$ is of the form $3k$

112 **(c)**

$2^{3n} - 1 = (2^3)^n - 1$

$= 8^n - 1 = (1+7)^n - 1$

$= 1 + {}^nC_1 7 + {}^nC_2 7^2 + \ldots + {}^nC_n 7^n - 1$

$= 7[{}^nC_1 + {}^nC_2 7 + ... + {}^nC_n 7^{n-1}]$

$\therefore 2^{3n} - 1$ is divisible by 7

113 **(b)**

We have,

$(\alpha - 2 + 1)^{35} = (1-\alpha)^{35}$

$\Rightarrow (\alpha - 1)^{35} = -(\alpha-1)^{35}$

$\Rightarrow 2(\alpha-1)^{35} = 0 \Rightarrow \alpha = 1$

114 **(b)**

We have,

$\therefore 3^{\log_3 \sqrt{25^{x-1}+7}}$ $\quad [\because a^{\log_a n} = n]$

$= \sqrt{25^{x-1}+7} = \sqrt{(5^{x-1}) + 7}$

$= \sqrt{y^2+7}$, where $y = 5^{x-1}$

and,

$3^{-(1/8)\log_3(5^{x-1}+1)}$

$= 3^{\log_3(5^{x-1}+1)^{-1/8}} = (5^{x-1}+1)^{-1/8} = (y+1)^{-1/8}$

$\therefore \left\{3^{\log_3\sqrt{25^{x-1}+7}} + 3^{-1/8\log_3(5^{x-1}+1)}\right\}^{10}$

$= \left[\sqrt{y^2+7} + (y+1)^{-1/8}\right]^{10}$

Now,

$T_9 = 180$

$\Rightarrow {}^{10}C_8 \left\{\left(\sqrt{y^2+7}\right)^{10-8} [(y+1)^{-1/8}\right\}^8 = 180$

$\Rightarrow {}^{10}C_8 (y^2+7)(y+1)^{-1} = 180$

$\Rightarrow 45\left(\frac{y^2+7}{y+1}\right) = 180$

$\Rightarrow y^2 + 7 = 4y + 4 \Rightarrow y^2 - 4y + 3 = 0$

$\Rightarrow y = 1, y = 3$

$\Rightarrow 5^{x-1} = 1$ or, $5^{x-1} = 3$

$\Rightarrow 5^x = 5$ or, $5^x = 15$

$\Rightarrow x = 1$ or, $x = \log_5 15$

$\Rightarrow x = \log_5 15 \quad [\because x > 1]$

115 **(c)**

The given sigma expansion

$\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} . 2^m$ can be written as

$[(x-3)+2]^{100} = (x-1)^{100} = (x-1)^{100}$

$\therefore$ Coefficient of x^{53} in $(1-x)^{100} =$

$(-1)^{53}\, {}^{100}C_{53} = -\,{}^{100}C_{53}$

116 **(c)**

The coefficient of x in the middle term of expansion of

$(1+\alpha x)^4 = {}^4C_2 \alpha^2$

The coefficient of x in the middle term of expansion of

$(1-\alpha x)^6 = {}^6C_3(-\alpha)^3$

Given, ${}^4C_2\alpha^2 = {}^6C_3(-\alpha)^3$

$\Longrightarrow \quad 6\alpha^2 = -20\alpha^3$

$\Longrightarrow \quad \alpha = \frac{-6}{20} = \frac{-3}{10}$

117 **(b)**

The general term in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ is

$T_{r+1} = {}^{10}C_r \left(\frac{x}{3}\right)^{\frac{10-r}{2}} \left(\frac{3}{2x^2}\right)^r$

$= {}^{10}C_r 3^{\frac{-10+3r}{2}} . 2^{-r} . x^{\frac{10-5r}{2}}$

For independent of x,

$\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$\therefore T_3 = {}^{10}C_2 \times \left(\frac{1}{3}\right)^4 \left(\frac{3}{2}\right)^2$

$= \frac{10 \times 9}{2 \times 1} \times \frac{1}{3 \times 3 \times 2 \times 2} = \frac{5}{4}$

118 **(a)**

Given, $(1 + x - 2x^2)^6 = 1 + a_1 x + a_2 x^2 + \ldots + a_{12} x^{12}$ ...(i)

On putting $x = 1$ in Eq. (i), we get

$(1+1-2)^6 = 1 + a_1 + a_2 + \cdots + a_{12}$

$\Rightarrow (0)^6 = 1 + a_1 + a_2 + \ldots + a_{12}$ ....(ii)

On putting $x = -1$ in Eq. (i), we get

$(1-1-2)^6 = 1 - a_1 + a_2 - a_3 + \cdots + a_{12}$

$\Rightarrow (-2)^6 = 1 + a_1 + a_2 - a_3 + \ldots + a_{12}$ ...(iii)

On adding Eqs. (ii) and (iii) we get

$(-2)^6 = 2(1 + a_2 + a_4 + \cdots + a_{12})$

$\Rightarrow \frac{64}{2} - 1 = a_2 + a_4 + \cdots + a_{12}$

$\therefore a_2 + a_4 + \cdots + a_{12} = 31$

119 **(c)**

Since, $(1 + x - 3x^2)^{10} = 1 + a_1 x + a_2 x^2 + \cdots + a_{20} x^{20}$

On putting $x = -1$, we get

$(1-1-3)^{10} = 1 - a_1 + a_2 - \cdots + a_{20}$

$= 3^{10}$ ... (i)

Again putting $x = 1$, we get

$(1+1-3)^{10} = 1 + a_1 + a_2 - \cdots + a_{20} = 1$ ... (ii)

On adding Eqs. (i) and (ii), we get

$2(1 + a_2 + a_4 + \cdots + a_{20}) = 3^{10} + 1$

$\Rightarrow a_2 + a_4 + \ldots + a_{20} = \frac{3^{10}+1}{2} - 1 = \frac{3^{10}-1}{2}$

120 **(b)**

We have,

$(1+x)^{2n} = (a_0 + a_2 x^2 + a_4 x^4 + \cdots) + x(a_1 + a_3 x^2 + a_5 x^4 + \cdots)$

Replacing x by i and $-i$ respectively and multiplying, we get

$(a_0 - a_2 + a_4 \ldots)^2 + (a_1 - a_3 + a_5 \ldots)^2$

$= (1+i)^{2n}(1-i)^{2n}$

$\Rightarrow (a_0 - a_2 + a_4 - \cdots)^2 + (a_1 - a_3 + a_5 \ldots)^2$

$= 2^{2n} = 4^n$

121 **(d)**

$(bc+ca+ab)^9=[bc+a(b+c)]^9$

$\therefore$ Coefficient of $a^5b^6c^7$

$=$ coefficient of $a^5b^6c^7$ in ${}^9C_5(bc)^4a^5(b+c)^5$

$=$coefficient of b^2c^3 *in* ${}^9C_5(b+c)^5$

$= {}^9C_5\times {}^5C_3=1260$

122 **(b)**

We have, $(x-1)(x-2)(x-3)\dots(x-100)$

Number of terms $=100$

$\therefore$ Coefficient of x^{99} in $(x-1)(x-2)(x-3)\dots(x-100)$

$=(-1-2-3-\dots-100)$

$=-(1+2+\dots+100)$

$=-\frac{100\times 101}{2}=-5050$

123 **(b)**

Given, $\sin n\theta=\sum_{r=0}^{n} b_r\sin^r\theta$

$\Rightarrow \sin n\theta=b_0\sin^0\theta+b_1\sin^1\theta+b_2\sin^2\theta+b_3\sin^3\theta+\dots+b_n\sin^n\theta$

$\Rightarrow \sin n\theta=b_0+b_1\sin\theta+b_2\sin^2\theta+\dots+b_n\sin^n\theta$

(n is an odd integer)

$\because \sin n\theta={}^nC_1\sin\theta\cos^{n-1}\theta-{}^nC_3\sin^3\theta\cos^{n-3}\theta+\dots$

$={}^nC_1\sin\theta(1-\sin^2\theta)^{(n-1)/2}-{}^nC_3\sin^3\theta(1-\sin^2\theta)^{(n-3)/2}+\dots$

$\therefore b_0=0, b_1=$ coefficient of $\sin\theta={}^nC_1=n$

($\because n-1, n-3$ are all even integers)

124 **(d)**

We have,

$\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$

$=2\left\{{}^6C_0x^6+{}^6C_2x^4\left(\sqrt{x^2-1}\right)^2+{}^6C_4x^2\left(\sqrt{x^2-1}\right)^4+{}^6C_6\left(\sqrt{x^2-1}\right)^6\right\}$

$=2\{x^6+{}^6C_2x^4(x^2-1)+{}^6C_2x^2(x^2-1)^2+(x^2-1)^3\}$

$=[\{2\ {}^6C_2+1\}x^6-3\{{}^6C_2+1\}x^4+4\ {}^6C_2x^2-1]$

Clearly, it contains 4 terms

125 **(a)**

We know that,

$(1+x)^n=C_0+C_1x+C_2x^2+\dots+C_nx^n$

$(x-1)^n=C_0x^n-C_1x^{n-1}+C_2x^{n-2}-\dots+(-1)^nC_n$

On multiplying both equations and equating the coefficient of x^n, we get

$C_0^2-C_1^2+C_2^2-\dots+(-1)^nC_n^2$

$={}^nC_{n/2}(-1)^{n/2}(x^2)^{n/2}$

Above is possible only when $\frac{n}{2}$ is an integer *ie*, n is even and in case n is odd, then term x^n will not occur

126 **(d)**

$(1-x+x^2)^n=a_0+a_1x+\dots+a_{2n}x^{2n}$

Putting $x=-1$ and 1

Successively and adding, we get

$a_0+a_2+a_4+\dots+a_{2n}=\frac{3^n+1}{2}$

127 **(d)**

Now, coefficient of x^{15} in $(1+x)^{20}$

$=$ coefficient of x^{15} in $(1+x)^{15}(1+x)^5$

$\Rightarrow {}^{20}C_{15}=$coefficient of x^{15} in $({}^{15}C_0x^{15}+{}^{15}C_1x^{14}+{}^{15}C_2x^{13}+{}^{15}C_3x^{12}+{}^{15}C_4x^{11}+{}^{15}C_5x^{10})$

$({}^5C_0x^5+{}^5C_1x^4+{}^5C_2x^3+{}^5C_3x^2+{}^5C_4x+{}^5C_5)$

$={}^{20}C_{15}={}^{15}C_0\cdot{}^5C_5+{}^{15}C_1\cdot{}^5C_4+{}^{15}C_2\cdot{}^5C_3+{}^{15}C_3\cdot{}^5C_2+{}^{15}C_4\cdot{}^5C_1+{}^{15}C_5\cdot{}^5C_0$

$\Rightarrow {}^{15}C_0\cdot{}^5C_5+{}^{15}C_1\cdot{}^5C_4+{}^{15}C_2\cdot{}^5C_3+{}^{15}C_3\cdot{}^5C_2+{}^{15}C_4\cdot{}^5C_1$

$={}^{20}C_{15}-{}^{15}C_5\cdot{}^5C_0$

$=\frac{20!}{5!15!}-\frac{15!}{5!10!}$

128 **(a)**

The given expression is

$1+(1+x)+(1+x)^2+\dots+(1+x)^n$ being in GP

Let $S=1+(1+x)+(1+x)^2+\dots+(1+x)^n$

$=\frac{(1+x)^{n+1}-1}{(1+x)-1}=x^{-1}[(1+x)^{n+1}-1]$

$\therefore$ The coefficient of x^k in S

$=$ The coefficient of x^k in $[(1+x)^{n+1}-1]$

$={}^{n+1}C_{k+1}$

129 **(d)**

Since, in a binomial expansion of $(a-b)^n, n\geq 5$, then sum of 5th and 6th terms is equal to zero.

$\therefore \quad {}^nC_4a^{n-4}(-b)^4+{}^nC_5a^{n-5}(-b)^5=0$

$\Rightarrow \frac{n!}{(n-4)!\,4!}a^{n-4}b^4-\frac{n!}{(n-5)!\,5!}a^{n-5}b^5=0$

$\Rightarrow \frac{n!}{(n-5)!\,4!}a^{n-5}\cdot b^4\left(\frac{a}{n-4}-\frac{b}{5}\right)=0$

$\Rightarrow \quad \frac{a}{b}=\frac{n-4}{5}$

130 **(a)**

We have,

$\left(1-\frac{1}{x}\right)^n(1-x)^n=(1-x)^{2n}\frac{(-1)^n}{x^n}$

$=\frac{(-1)^n(1-x)^{2n}}{x^n}$

$\therefore$ Middle term in $\left(1-\frac{1}{x}\right)^n(1-x)^n$

$=\frac{(-1)^n}{x^n}$ middle term in $(1-x)^{2n}$

$=\frac{(-1)^n}{x^n}\times(n+1)^{th}$ term in $(1-x)^{2n}$

$=\frac{(-1)^n}{x^n}\times{}^{2n}C_n(-x)^n={}^{2n}C_n$

132 **(d)**

The number of terms in the expansion of $(a+b+c)^{10}$

$={}^{12}C_2=\frac{11\cdot 12}{2}=66$

133 **(d)**

The given expression of $\frac{1}{(4-3x)^{1/2}}$ can be rewritten as

$4^{-1/2}\left(1-\frac{3}{4}x\right)^{-1/2}$ and it is valid only when

$\left|\frac{3}{4}x\right|<1$

$\Rightarrow -\frac{4}{3}<x<\frac{4}{3}$

134 **(c)**

$\because (3+2x)^{50}=3^{50}\left(1+\frac{2x}{3}\right)^{50}$

Here, $T_{r+1}=3^{50}\,{}^{50}C_r\left(\frac{2x}{3}\right)^r$

and $T_r=3^{50}\,{}^{50}C_{r-1}\left(\frac{2x}{3}\right)^{r-1}$

But $x=\frac{1}{5}$

$\therefore \frac{T_{r+1}}{T_r}\geq 1\Rightarrow \frac{{}^{50}C_r}{{}^{50}C_{r-1}}\frac{2}{3}\cdot\frac{1}{5}\geq 1$

$\Rightarrow 102-2r\geq 15r \Rightarrow r\leq 6$

135 **(a)**

Given that, $(1+ax)^n=1+8x+24x^2+\dots$

$\Rightarrow 1+\frac{n}{1}ax+\frac{n(n-1)}{1.2}a^2x^2+\dots$

$=1+8x+24x^2+\dots$

On comparing the coefficients of x, x^2, we get

$na=8, \frac{n(n-1)}{1.2}a^2=24$

$\Rightarrow na(n-1)a=48$

$\Rightarrow 8(8-a)=48$

$\Rightarrow 8-a=6$

$\Rightarrow a=2 \Rightarrow n=4$

136 **(b)**

$\because (0.99)^{15}=(1-0.01)^{15}$

$=1-{}^{15}C_1(0.01)+{}^{15}C_2(0.01)^2$

$-{}^{15}C_3(0.01)^3+\dots$

We want to answer correct upto 4 decimal places and as such, we have left further expansion.

$=1-15(0.01)+\frac{15\cdot 14}{1\cdot 2}(0.0001)$

$-\frac{15\cdot 14\cdot 13}{1\cdot 2\cdot 3}(0.000001)+\dots$

$=1-0.15+0.0105-0.000455+\dots$

$=1.0105-0.150455$

$=0.8601$

137 **(b)**

Given that,

$\frac{1}{1!\,(n-1)!}+\frac{1}{3!\,(n-3)!}+\frac{1}{5!\,(n-5)!}+\dots$

$=\frac{1}{n!}\left[\frac{n!}{1!\,(n-1)!}+\frac{n!}{3!\,(n-3)!}+\frac{n!}{5!\,(n-5)!}+\dots\right]$

$=\frac{1}{n!}[{}^nC_1+{}^nC_3+{}^nC_5+\dots]$

$=\frac{2^{n-1}}{n!}$

138 **(b)**

$\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$

$=\frac{\left(1+\frac{3}{2}x+\frac{\frac{3}{2}\frac{1}{2}}{2}x^2\right)-\left(1+\frac{3x}{2}+\frac{3\cdot 2}{2}\cdot\frac{x^2}{4}\right)}{(1-x)^{1/2}}$

[neglecting higher powers of x]

$=-\frac{3x^2}{8}(1-x)^{-1/2}$

$=-\frac{3x^2}{8}\left(1+\frac{1}{2}x+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2}\cdot x^2\right)=-\frac{3x^2}{8}$

[neglecting higher powers of x]

139 **(c)**

Total number of terms in the expansion of $(2x+3y-4z)^n$, is

${}^{n+3-1}C_{3-1}={}^{n+2}C_2=\frac{(n+2)(n+1)}{2}$

140 **(b)**

We have,

$(1+x)^m(1+x)^n=\left(\sum_{r=0}^{m}{}^mC_r\,x^r\right)\cdot\left(\sum_{r=0}^{n}{}^nC_r\,x^r\right)$

Equation coefficients of x^r on both sides, we get

${}^mC_r+{}^mC_{r-1}\,{}^nC_1+{}^mC_{r-2}\,{}^nC_2+\dots+{}^mC_1\,{}^nC_{r-1}$

$+\dots+{}^mC_0\,{}^nC_r={}^{m+n}C_r$

141 **(c)**

$(1-a\,x)^{-1}(1-bx)^{-1}$

$=(a^0+a\,x+a^2x^2+\dots(b^0+bx+b^2x^2+\dots)$

Hence, $a_n=$coefficient of x^n in $(1-ax)^{-1}(1-bx)^{-1}$

$a^0b^n+ab^{n-1}+\dots+a^nb^0$

$= a^0 b^n \left(1 + \frac{a}{b} + \left(\frac{a}{b}\right)^2 + \dots + \left(\frac{a}{b}\right)^n\right)$

$= a^0 b^n \left(\frac{\left(\frac{a}{b}\right)^{n+1} - 1}{\frac{a}{b} - 1}\right)$

$= \frac{a^{n+1} - b^{n+1}}{a - b} = \frac{b^{n+1} - a^{n+1}}{b - a}$

142 **(c)**

Given, $(1 + 2x + x^2)^5 = \sum_{k=0}^{15} a_k x^k$

$\Rightarrow (1 + x)^{10} = a_0 x^0 + a_1 x + a_2 x^2 + \dots + a_{15} x^{15}$

$\Rightarrow {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + \dots + {}^{10}C_{10} x^{10}$

$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_{15} x^{15}$

On equating the coefficient of constant and even power of x, we get

$a_0 = 10_{C_0}, a_2 = 10_{C_2},$

$a_4 = {}^{10}C_4, \dots a_{10} = {}^{10}C_{10}, a_{12} = a_{14} = 0$

$\therefore \sum_{k=0}^{7} a_{2k} = {}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + {}^{10}C_6$

$+ {}^{10}C_8 + {}^{10}C_{10} + 0 + 0$

$= 2^{10-1} = 2^9 = 512$

143 **(b)**

Since, n is even, therefore $\left(\frac{n}{2} + 1\right)$th term is the middle term.

$\therefore T_{\frac{n}{2}+1} = {}^nC_{n/2} (x^2)^{n/2} \left(\frac{1}{x}\right)^{n/2}$

$= 924x^6$ (given)

$\Rightarrow x^{n/2} = x^6 \Rightarrow n = 12$

144 **(b)**

We have, $(1 + x^2)^5 (1 + x)^4$

$= ({}^5C_0 + {}^5C_1 x^2 + {}^5C_2 x^4 + \dots)({}^4C_0 + {}^4C_1 x + {}^4C_2 x^2 + {}^4C_3 x^3 + {}^4C_4 x^4)$

The coefficient of x^5 in $[(1 + x^2)^5 (1 + x)^4]$

$= {}^5C_2 . {}^4C_1 + {}^4C_3 . {}^5C_1 = 10.4 + 4.5 = 60$

145 **(d)**

$(1 + x + x^2 + x^3)^n = \{(1 + x)^n (1 + x^2)^n\}$

$= (1 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n)(1 + {}^nC_1 x^2 + {}^nC_2 x^4 + \dots + {}^nC_n x^{2n})$

Therefore the coefficient of $x^4 = {}^nC_2 + {}^nC_2 \, {}^nC_1 + {}^nC_4$

$= {}^nC_4 + {}^nC_2 + {}^nC_1 \, {}^nC_2$

146 **(b)**

Let $a = {}^nC_{r-1}, b = {}^nC_r, c = {}^nC_{r+1}$

and $d = {}^nC_{r+2}$

$\therefore a + b = {}^{n+1}C_r, b + c = {}^{n+1}C_{r+1}, c + d = {}^{n+1}C_{r+2}$

$\Rightarrow \frac{a + b}{a} = \frac{{}^{n+1}C_r}{{}^nC_{r-1}} = \frac{n + 1}{r} \Rightarrow \frac{a}{a + b} = \frac{r}{n + 1}$

and $\frac{b+c}{b} = \frac{{}^{n+1}C_{r+1}}{{}^nC_r} = \frac{n+1}{r+1} \Rightarrow \frac{b}{b+c} = \frac{r+1}{n+1}$

and $\frac{c+d}{c} = \frac{{}^{n+1}C_{r+2}}{{}^nC_{r+1}} = \frac{n+1}{r+2} \Rightarrow \frac{c}{c+d} = \frac{r+2}{n+1}$

$\therefore \frac{a}{a+b}, \frac{b}{b+c}, \frac{c}{c+d}$ are in AP

$\because$ AM $> GM$

$\Rightarrow \frac{b}{b + c} > \sqrt{\frac{ac}{(a + b)(c + d)}}$

or $\left\{\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right\} > 0$

147 **(d)**

We have,

$T_{r+1} = {}^6C_r \left(\sqrt{x^5}\right)^{6-r} \left(\frac{3}{\sqrt{x^3}}\right)^r$

$\Rightarrow T_{r+1} = {}^6C_r \, x^{15 - \frac{5}{2}r - \frac{3}{2}r} 3^r = {}^6C_r \, x^{15-4r} 3^r$

This will contain x^3, if $15 - 4r = 3 \Rightarrow r = 3$

$\therefore$ Coefficient of $x^3 = {}^6C_3 \cdot 3^3 = 540$

148 **(b)**

General term, $T_{r+1} = {}^{11}C_r \frac{a^{11-r}}{b^r} (-1)^r x^{11-3r}$

For the coefficient of x^{-7}, put

$11 - 3r = -7 \Rightarrow r = 6$

$\therefore$ Coefficient of $x^{-7} = {}^{11}C_6 \frac{a^5}{b^6} = \frac{462a^5}{b^6}$

149 **(a)**

We have,

Coefficient of x^5 in $(x + 3)^6 = {}^6C_1 \times 3^1 = 18$

150 **(d)**

$A_r =$ Coefficient of x^r in $(1 + x)^{10} = {}^{10}C_r$

$B_r =$ Coefficient of x^r in $(1 + x)^{20} = {}^{20}C_r$

$C_r =$ Coefficient of x^r in $(1 + x)^{30} = {}^{30}C_r$

$\therefore \sum_{r=1}^{10} A_r (B_{10} B_r - C_{10} A_r)$

$= \sum_{r=1}^{10} A_r B_{10} B_r, - \sum_{r=1}^{10} A_r C_{10} A_r$

$= \sum_{r=1}^{10} {}^{10}C_r \, {}^{20}C_{10} \, {}^{20}C_r \sum_{r=1}^{10} {}^{10}C_r \, {}^{30}C_{10} \, {}^{10}C_r l$

$\sum_{r=1}^{10} {}^{10}C_{10-rl} \, {}^{20}C_{10} \, {}^{20}C_r - \sum_{r=1}^{10} {}^{10}C_{10-r} \, {}^{30}C_{10} \, {}^{10}C_r l$

$= {}^{20}C_{10} \sum_{r=1}^{10} {}^{10}C_{10-r} \, {}^{20}C_r$

$- {}^{30}C_{10} \sum_{r=1}^{10} {}^{10}C_{10-r} \, {}^{10}C_r$

$= {}^{20}C_{10}({}^{30}C_{10}-1)-{}^{30}C_{10}({}^{20}C_{10}-1)$

$= {}^{20}C_{10}({}^{30}C_{10}-1)-{}^{30}C_{10}({}^{20}C_{10}-1)$

$= {}^{30}C_{10}-{}^{20}C_{10}=C_{10}-B_{10}$

151 **(a)**

$\because$Coefficient of $x^p$is ${}^{p+q}C_p$ and coefficient of x^q is ${}^{(p+q)}C_q$

$\therefore$ Both the coefficients are equal

152 **(c)**

In the expansion of $(1+x)^{2n}$, the general term $= {}^{2n}C_k x^k, 0\le k\le 2n$

As given for $r>1, n>2$, ${}^{2n}C_{3r}={}^{2n}C_{r+2}$

$\Rightarrow$Either $3r=r+2$ or $3r=2n-(r+2)$ $(\because {}^nC_r={}^nC_{n-r})$

$\Rightarrow r=1$ or $n=2r+1$

We take the relation only

$n=2r+1$ $(\because r>1)$

153 **(a)**

The general term in the expansion of $\left(x\sin^{-1}\alpha+\frac{\cos^{-1}\alpha}{x}\right)^{10}$ is given by

$$T_{r+1}={}^{10}C_r(x\sin^{-1}\alpha)^{10-r}\left(\frac{\cos^{-1}\alpha}{x}\right)^r$$

$$\Rightarrow T_{r+1} = {}^{10}C_r(\sin^{-1}\alpha)^{10-r}(\cos^{-1}\alpha)^r x^{10-2r} \quad \dots\text{(i)}$$

This will be independent of x, if

$10-2r=0\Rightarrow r=5$

Putting $r=5$ in (i), we get

$$T_6={}^{10}C_5(\sin^{-1}\alpha\cos^{-1}\alpha)^5$$

$$\Rightarrow T_6={}^{10}C_5\left\{\sin^{-1}\alpha\left(\frac{\pi}{2}-\sin^{-1}\alpha\right)\right\}^5$$

$$\Rightarrow T_6={}^{10}C_5\left\{\frac{\pi}{2}\sin^{-1}\alpha-(\sin^{-1}\alpha)^2\right\}^5$$

$$\Rightarrow T_6={}^{10}C_5\left\{\frac{\pi^2}{16}-\left(\frac{\pi}{4}-\sin^{-1}\alpha\right)^2\right\}^5$$

Now,

$$-\frac{\pi}{2}\le\sin^{-1}\alpha\le\frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{2}\le-\sin^{-1}\alpha\le\frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4}\le\left(\frac{\pi}{4}-\sin^{-1}\alpha\right)\le\frac{3\pi}{4}$$

$$\Rightarrow 0\le\left(\frac{\pi}{4}-\sin^{-1}\alpha\right)^2\le\frac{9\pi^2}{16}$$

$$\Rightarrow -\frac{9\pi^2}{16}\le-\left(\frac{\pi}{4}-\sin^{-1}\alpha\right)^2\le 0$$

$$\Rightarrow -\frac{\pi^2}{2}\le\frac{\pi^2}{16}-\left(\frac{\pi}{4}-\sin^{-1}\alpha\right)^2\le\frac{\pi^2}{16}$$

$$\Rightarrow -{}^{10}C_5\left(\frac{\pi^2}{2}\right)^5\le{}^{10}C_5\left\{\frac{\pi^2}{16}-\left(\frac{\pi}{4}-\sin^{-1}\alpha\right)^2\right\}^5 \le {}^{10}C_5\left(\frac{\pi^2}{16}\right)^5$$

$$\Rightarrow -\frac{{}^{10}C_5\,\pi^{10}}{2^5}\le T_6\le{}^{10}C_5\frac{\pi^{10}}{2^{20}}$$

154 **(b)**

In the expansion of $(3+7x)^{29}$

$$T_{r+1}={}^{29}C_r\cdot 3^{29-r}\cdot(7x)^r$$
$$=({}^{29}C_r\times 3^{29-r}\times 7^r)x^r$$

Let a_r= coefficient of $(r+1)$th term

$$={}^{29}C_r\times 3^{29-r}\times 7^r$$

and a_{r-1} =coefficient of rth term

$$={}^{29}C_{r-1}\times 3^{30-r}\times 7^{r-1}$$

According to question $a_r=a_{r-1}$

$$\Longrightarrow {}^{29}C_r\times 3^{29-r}\times 7^r={}^{29}C_{r-1}\times 3^{30-r}\times 7^{r-1}$$

$$\Longrightarrow \frac{{}^{29}C_r}{{}^{29}C_{r-1}}=\frac{3}{7}\Longrightarrow\frac{30-r}{r}=\frac{3}{7}$$

$$\Longrightarrow 210-7r=3r\Longrightarrow r=21$$

156 **(b)**

In the expansion of $(1+x)^{50}$ the sum of the coefficient of odd powers

$=C_1+C_3+C_5+\ldots=2^{50-1}=2^{49}$

157 **(b)**

It is given that the coefficients of r^{th} and $(r+1)^{\text{th}}$ term in the expansion of $(3+7x)^{29}$ are equal

$$\therefore {}^{29}C_{r-1}\times 3^{30-r}\times 7^{r-1}={}^{29}C_r\times 3^{29-r}\times 7^r$$

$$\Rightarrow {}^{29}C_{r-1}\times 3={}^{29}C_r\times 7$$

$$\Rightarrow\frac{3}{30-r}=\frac{7}{r}\Rightarrow r=21$$

158 **(b)**

We have,

$${}^{2n}C_p={}^{2n}C_{p+2}\Rightarrow p+p+2=2n\Rightarrow p=n-1$$

159 **(b)**

We have

$$(1+x)^n=\sum_{r=0}^{n}a_r x^r\Rightarrow a_r={}^nC_r$$

Now,

$$\left(1+\frac{a_1}{a_0}\right)\left(1+\frac{a_2}{a_1}\right)\ldots\left(1+\frac{a_n}{a_{n-1}}\right)$$

$$=\prod_{r=1}^{n}\left(1+\frac{a_r}{a_{r-1}}\right)$$

$$=\prod_{r=1}^{n}\left(\frac{a_{r-1}+a_r}{a_{r-1}}\right)$$

$$=\prod_{r=1}^{n}\left(\frac{{}^nC_r+{}^nC_{r-1}}{{}^nC_{r-1}}\right)$$

$= \prod_{r=1}^{n} \frac{{}^{n+1}C_r}{{}^{n}C_{r-1}}$

$= \prod_{r=1}^{n} \frac{n+1}{r}$ $\left[\because {}^{n+1}C_r = \frac{n+1}{r}\, {}^{n}C_{r-1}\right]$

$= (n+1)^n \left(\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}\times \ldots \times\frac{1}{n}\right) = \frac{(n+1)^n}{n\,!}$

161 **(a)**

$(2x^2 - x - 1)^5 = a_0 + a_1x + a_2x^2 + \cdots + a_{10}x^{10}$

On putting $x = 0$, we get

$-1 = a_0$

On putting $x = 1$, we get

$0 = a_0 + a_1 + a_2 + \ldots + a_{10}$ ...(i)

On putting $x = -1$, we get

$(2+1-1)^5 = a_0 - a_1 + a_2 - \ldots + a_{10}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$0 + (2)^5 = 2(a_0 + a_2 + \cdots + a_{10})$

$\Rightarrow \quad 16 - 1 = a_2 + \cdots + a_{10}$

$\Rightarrow a_2 + a_3 + \ldots + a_{10} = 15$

162 **(b)**

We have,

${}^{2n}C_r = {}^{2n}C_{r+2} \Rightarrow r + r + 2 = 2n \Rightarrow n = r + 1$

163 **(d)**

$\therefore$ coefficient of x^{100} in the expansion of $\sum_{j=0}^{200}(1+x)^j$ will be $\sum_{j=0}^{200}\ {}^{j}C_{100}$

$= [{}^{100}C_{100} + {}^{101}C_{100} + {}^{102}C_{100} + \cdots + {}^{200}C_{100}]$

$[\because {}^{n}C_n + {}^{n+1}C_n + {}^{n+2}C_n + \cdots + {}^{2n-1}C_n = {}^{2n}C_{n+1}]$

$= \binom{201}{100}$

164 **(b)**

We have,

$\frac{1}{n\,!} + \frac{1}{2\,!\,(n-2)\,!} + \frac{1}{4\,!\,(n-4)\,!} + \cdots$

$= \frac{1}{n\,!}\left\{\frac{n\,!}{n\,!} + \frac{n\,!}{2\,!\,(n-2)\,!} + \frac{n\,!}{4'!\,(n-4)\,!} + \cdots\right\}$

$= \frac{1}{n\,!}\{{}^{n}C_0 + {}^{n}C_2 + {}^{n}C_4 + \cdots\} = \frac{2^{n-1}}{n\,!}$

165 **(c)**

General term $T_{r+1} = {}^{10}C_r \left(\frac{x}{2}\right)^{10-r}\left(-\frac{3}{x^2}\right)^r$

$= {}^{10}C_r \cdot \frac{x^{10-3r}\cdot(-1)^r\cdot 3^r}{2^{10-r}}$

For the coefficient of x^4 put

$10 - 3r = 4$

$\Rightarrow \quad r = 2$

Hence, coefficient of x^4 is

${}^{10}C_2 \cdot \frac{3^2}{2^8} = \frac{405}{256}$

166 **(a)**

Given, $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$

Also, $(x+1)^n = C_n + C_{n-1}x + C_{n-2}x^2 + \cdots + C_0x^n$

On multiplying both equations and comparing coefficient of x^{n-1} on both sides, we get

$C_0C_2 + C_1C_2 + C_2C_3 + \cdots + C_{n-1}C_n = {}^{2n}C_{n-1}$

$= \frac{(2n)!}{(n-1)!\,(n+1)!}$

167 **(c)**

Now, $7^9 = (8-1)^9 = -1(1-8)^9$

$= -1 + {}^{9}C_1 8 - {}^{9}C_2 8^2 + \ldots + {}^{9}C_9 8^9$ and $9^7 = (1+8)^7$

$= 1 + {}^{7}C_1 8 + {}^{7}C_2 8^2 + {}^{7}C_3 8^3 + \ldots + {}^{7}C_7 8^7$

$\therefore\ 7^9 + 9^7 = 8({}^{9}C_1 + {}^{7}C_1) + 8^2({}^{7}C_2 - {}^{9}C_2) + \cdots$

$= 8(9+7) + 8^2(21-36) + \cdots$

$= 64 \times 2 + 64(-15) + \cdots$

Hence, it is divisible by 64

168 **(d)**

Let $f = \left(8 - 3\sqrt{7}\right)^{10}$, here $0 < f < 1$

$\therefore\ \left(8+3\sqrt{7}\right)^{10} + \left(8-3\sqrt{7}\right)^{10}$ is an integer hence, this is the value of n

169 **(d)**

We have, $\left(3x - \frac{1}{2x}\right)^8$

$\therefore$ Ninth term $T_9 = 8_{c_8}(3x)^{8-8}\left(\frac{-1}{2x}\right)^8$

$= \frac{1}{256x^8}$

170 **(b)**

The general term in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$ is given by

$T_{r+1} = {}^{n-3}C_r (x)^{n-3-r}\left(\frac{1}{x^2}\right)^r$

$= {}^{n-3}C_r x^{n-3-3r}$

As x^{2k} occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$, we must have $n - 3 - 3r = 2k$ for some non-negative integer r

$\Rightarrow\ 3(1+r) = n - 2k$

$\Rightarrow\ n - 2k$ is a multiple of 3

171 **(c)**

Let T_{r+1} denote the $(r+1)^{\text{th}}$ term in the expansion of $\left(7^{1/3} + 5^{1/2}x\right)^{600}$. Then,

$T_{r+1} = {}^{600}C_r\,(7^1)^{600-r}\left(5^{1/3}x\right)^r$

$= {}^{600}C_r 7^{200-\frac{r}{3}} \times 5^{\frac{r}{2}} \times x^r$

Here, $0 \le r \le 600$

For $200 - \frac{r}{3}$ and $\frac{r}{2}$ to be integers, we must have

$\frac{r}{3}$ and $\frac{r}{2}$ as integers, and $0 \le r \le 600$

$\Rightarrow r$ is multiple of 2 and 3 both and $0 \le r \le 600$

$\Rightarrow r$ is a multiple of 6 and $0 \le r \le 600$

$\Rightarrow r = 0,6,12, \dots ,600$

Hence, there are 101 terms with integral coefficients

172 **(b)**

We have,

$$(xy + yz + zx)^6 = \sum_{r+s+t=6} \frac{6!}{r!\,s!\,t!}(xy)^r(yz)^s(zx)^t$$

$$= \sum_{r+s+t=6} \frac{6!}{r!\,s!\,t!} x^{r+t}y^{r+s}z^{s+t}$$

If the general term in the above expansion contains $x^3y^4z^5$, then

$r + t = 3, r + s = 4$ and $s + t = 5$

Also, $r + s + t = 6$

On solving these equations, we get

$r = 1, s = 3, t = 2$

$\therefore$ Coefficient of $x^3y^4z^5 = \frac{6!}{1!\,3!\,2!} = 60$

173 **(c)**

We have,

$$(1 + 2x + x^2)^n = \sum_{r=0}^{2n} a_r\, x^r$$

$$\Rightarrow \{(1 + x)^2\}^n = \sum_{r=0}^{2n} a_r\, x^r$$

$$\Rightarrow (1 + x)^{2n} = \sum_{r=0}^{2n} a_r\, x^r$$

$$\Rightarrow \sum_{r=0}^{2n} {}^{2n}C_r\, x^r = \sum_{r=0}^{2n} a_r\, x^r \Rightarrow a_r = {}^{2n}C_r$$

174 **(a)**

Given, ${}^{20}C_4 + {}^{20}C_3 + {}^{20}C_3 + {}^{20}C_2 - {}^{22}C_{18}$

$= {}^{21}C_4 + {}^{20}C_3 + {}^{20}C_2 - {}^{22}C_{18}$

$= {}^{22}C_4 - {}^{22}C_{18} = {}^{22}C_{18} - {}^{22}C_{18} = 0$

175 **(d)**

We have,

$y = 3\,x + 6\,x^2 + 10\,x^3 + \cdots$

$\Rightarrow 1 + y = (1 + 3\,x + 6\,x^2 + 10\,x^3 + \cdots)$

$\Rightarrow 1 + y = (1 - x)^{-3}$

$\Rightarrow (1 - x) = (1 + y)^{-1/3}$

$\Rightarrow x = 1 - (1 + y)^{-1/3}$

$\Rightarrow x = \frac{1}{3}\, y - \frac{1 \cdot 4}{3^2 \cdot 2} y^2 + \frac{1 \cdot 4 \cdot 7 \cdot}{3^3\, 3\,!} y^3 \dots$

176 **(c)**

We know that,

$(x + a)^n + (x - a)^n = 2[\,{}^nC_0x^n + {}^nC_2x^{n-2}a^2 + \dots]$

Here, $n = 5, x = x$ and $a = (x^3 - 1)^{1/2}$

$\therefore [x + (x^3 - 1)^{1/2}]^5[x - (x^3 - 1)^{1/2}]^5$

$= 2[\,{}^5C_0x^5 + {}^5C_2x^3(x^3 - 1) + {}^5C_4x(x^3 - 1)^2]$

$= 2[x^5 + 10x^3(x^3 - 1) + 5\,x(x^3 - 1)^2]$

$\therefore$ Given expression is a polynomial of degree 7.

177 **(c)**

We have,

$C_0^2 + 3 \cdot C_1^2 + 5 \cdot C_2^2 + \cdots + (2n + 1)C_n^2$

$= \{C_0^2 + C_1^2 + C_2^2 + \cdots + C_n^2\}$

$+\{2\,C_1^2 + 4 \cdot C_2^2 + 6 \cdot C_3^2 + \cdots + 2\,n\,C_n^2\}$ ... (i)

We have,

$(1 + x)^{2n} = (1 + x)^n(1 + x)^n$

$\Rightarrow (1 + x)^{2n} = (C_0 + C_1x + C_2x^2 + \cdots + C_nx^n) \times (C_0x^n + C_1x^{n-1} + \cdots + C_{n-1}\,x + C_n)$

On equating the coefficient of x^n on both sides, we get

${}^{2n}C_n = C_0^2 + C_1^2 + C_2^2 + \cdots C_n^2$ ... (ii)

Also,

$n(1 + x)^{n-1}(1 + x)^n = (C_1 + 2\,C_2\,x + 3\,C_3\,x^2 + \cdots + n\,C_n\,x^{n-1}) \times (C_0\,x^n + C_1\,x^{n-1} + C_2\,x^{n-2} + \cdots + C_n)$

On equating the coefficient of x^{n-1} on both sides, we get

$n \cdot {}^{2n-1}C_{n-1} = (C_1^2 + 2\,C_2^2 + 3\,C_3^2 + \cdots + n\,C_n^2)$

$\Rightarrow 2n \cdot {}^{2n-1}C_{n-1} = 2\,C_1^2 + 4\,C_2^2 + 6\,C_3^2 + \cdots + +2\,n\,C_n^2$ ... (iii)

From (i),(ii) and (iii), we obtain

$C_0^2 + 3 \cdot C_1^2 + 5\,C_2^2 + \cdots + (2n + 1)C_n^2$

$= \frac{2n}{n}\,{}^{2n-1}C_{n-1} + 2n \cdot {}^{2n-1}C_{n-1}$

$= 2(n + 1)\,{}^{2n-1}C_{n-1}$

178 **(d)**

Here ${}^{n-1}C_r = (k^2 - 3)\,{}^nC_{r+1}$

$\Rightarrow {}^{n-1}C_r = (k^2 - 3)\frac{n}{r + 1}\,{}^{n-1}C_r$

$\Rightarrow k^2 - 3 = \frac{r + 1}{n}$

$\left[\text{since}, n - 1 \ge r \Rightarrow \frac{r + 1}{n} \le 1 \text{and } n, r \ge 0\right]$

$\Rightarrow 0 < k^2 - 3 \le 1 \Rightarrow 3 < k^2 \le 4$

$\Rightarrow k \in \left[-2, -\sqrt{3}\right) \cup \left(\sqrt{3}, 2\right]$

179 **(a)**

Let $f(x) = (x + y)^{100} + (x - y)^{100}$

Here, $n = 100$, which is even.

$\therefore$ Total number of terms

$= \frac{n+2}{2} = \frac{100+2}{2}$
$= 51$

180 **(c)**

We know that

$C_0^2 + C_1^2 + C_2^2 + C_3^2 + \cdots + C_n^2 = \frac{(2n)!}{n!\,n!}$ ...(i)

and, $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \cdots - C_n^2 = 0$, when n is odd ...(ii)

subtracting (ii) from (i), we get

$2(C_1^2 + C_3^2 + C_5^2 + \cdots + C_n^2) = \frac{(2n)!}{(n!)^2}$

$\Rightarrow C_1^2 + C_3^2 + C_5^2 + \cdots + C_n^2 = \frac{(2n)!}{2(n!)^2}$

181 **(a)**

$\sum_{k=0}^{10} {}^{20}C_k = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_{10}$

On putting $x = 1$and $n = 20$in $(1+x)^n$

$= {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_n x^n$

We get

$2^{20} = 2({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9) + {}^{20}C_{10}$

$\Longrightarrow 2^{19} = ({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9) + \frac{1}{2}\,{}^{20}C_{10}$

$\Longrightarrow 2^{19} = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_{10} - \frac{1}{2}\,{}^{20}C_{10}$

$\Longrightarrow {}^{20}C_0 + {}^{20}C_1 + \ldots + {}^{20}C_{10} = 2^{19} + \frac{1}{2}\,{}^{20}C_{10}$

182 **(b)**

$(7.995)^{1/3} = (8 - 0.005)^{1/3}$

$= (8)^{1/3}\left[1 - \frac{0.005}{8}\right]^{1/3}$

$= 2\left[1 - \frac{1}{3} \times \frac{0.005}{8} + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2 \cdot 1}\left(\frac{0.005}{8}\right)^2 + \ldots\right]$

$= 2\left[1 - \frac{0.005}{24} - \frac{\frac{1}{3} \times \frac{1}{3}}{1} \times \frac{(0.005)^2}{64} + \ldots\right]$

$= 2(1 - 0.000208)$ (neglecting other terms)

$= 2 \times 0.999792$

$= 1.9996$

183 **(b)**

It is given that ${}^mC_1$, ${}^mC_2$ and ${}^mC_3$ are in A.P.

$\therefore 2\,{}^mC_2 = {}^mC_1 + {}^mC_3$

$\Rightarrow m^2 - 9m + 14 = 0$

$\Rightarrow m = 2, 7$

For $m = 2$, there are only three terms. Therefore, $m = 7$.

Now,

$\Rightarrow 21 = {}^7C_5 \left\{\sqrt{2^{\log_{10}(10-3^x)}}\right\}^{7-5} \left\{\sqrt[5]{2^{(x-2)\log_{10}3}}\right\}^5$

$\Rightarrow 21 = 21 \cdot 2^{\log_{10}(10-3^x)} \cdot 2^{(x-2)\log_{10}3}$

$\Rightarrow 1 - 2^{\log_{10}(10-3^x)+(x-2)\log_{10}3}$

$\Rightarrow 2^0 = 2^{\log_{10}[(10-3^x)\cdot 3^{x-2}]}$

$\Rightarrow (10 - 3^x)3^{x-2} = 1$

$\Rightarrow 3^{2x-2} - 10.3^{x-2} + 1 = 0$

$\Rightarrow 3^{2x} - 10.3^x + 9 = 0$

$\Rightarrow (3^x - 1)(3^x - 9) = 0$

$\Rightarrow 3^x = 1, 3^x = 9 \Rightarrow x = 0, 2$

184 **(b)**

Given, ${}^nC_{12} = {}^nC_6$

or ${}^nC_{n-12} = {}^nC_6$

$\Longrightarrow n - 12 = 6 \Longrightarrow n = 18$

$\therefore {}^nC_2 = {}^{18}C_2 = 153$

185 **(b)**

Let $(r+1)$th term be the coefficient of x^0 in the expansion of

$\left(x - \frac{1}{x}\right)^6$.

$\therefore T_{r+1} = {}^6C_r x^{6-r}\left(-\frac{1}{x}\right)^r$

$= (-1)^r\, {}^6C_r x^{6-2r}$

Since, this term is a constant term.

$\therefore 6 - 2r = 0 \Longrightarrow r = 3$

$\therefore T_4 = (-1)^3\, {}^6C_3 = -20$

186 **(d)**

General term, $T_{r+1} = {}^{15}C_r (x^3)^{15-r}\left(\frac{2}{x^2}\right)^r$

$= {}^{15}C_r x^{45-5r}(2)^r$

For term independent of x, put $45 - 5r = 0 \Longrightarrow r = 9$

$\therefore$ Independent term $= T_{9+1} = T_{10}$

187 **(b)**

We have, $T_{r+1} = {}^{21}C_r \left(\frac{a^{1/3}}{b^{1/6}}\right)^{21-r}\left(\frac{b^{1/2}}{a^{1/6}}\right)^r$

$= {}^{21}C_r \frac{a^{7-(r/3)}}{b^{7/2-r/6}} \cdot \frac{b^{r/2}}{a^{r/6}}$

$= {}^{21}C_r a^{7-(r/2)} b^{2r/3-7/2}$

Since, exponents of a and b in the $(r+1)$th term are equal

$\therefore 7 - \frac{r}{2} = \frac{2r}{3} - \frac{7}{2}$

$\Rightarrow \frac{21}{2} = \frac{7}{6}r \Rightarrow r = 9$

188 **(b)**

$\left(\frac{1}{x} + 1\right)^n (1+x)^n = \frac{1}{x^n}(1+x)^{2n}$

$$= \frac{1}{x^n}(1 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \cdots + {}^{2n}C_{n-1}x^{n-1} + \cdots + {}^{2n}C_{2n}x^{2n})$$

The coefficient of $\frac{1}{x}$ is ${}^{2n}C_{n-1}$.

189 **(d)**

$$x = (\sqrt{3}+1)^5 = (\sqrt{3})^5 + {}^5C_1(\sqrt{3})^4 + {}^5C_2(\sqrt{3})^3 + {}^5C_3(\sqrt{3})^2 + {}^5C_4(\sqrt{3}) + {}^5C_5$$

$$= 9\sqrt{3} + 45 + 30\sqrt{3} + 30 + 5\sqrt{3} + 1$$

$$= 76 + 44\sqrt{3}$$

$$\therefore [x] = \left[(\sqrt{3}+1)^5\right] = [76 + 44\sqrt{3}]$$

$$= [76] + [44 \times 1.732]$$

$$= 76 + [76.2]$$

$$= 76 + 76 = 152$$

190 **(a)**

We have,

$$2\,C_0 + \frac{2^2}{2}C_1 + \frac{2^3}{2}C_2 + \cdots + \frac{2^{11}}{11}C_{10}$$

$$= \sum_{r=0}^{10} {}^{10}C_r \frac{2^{r+1}}{r+1}$$

$$= \frac{1}{11}\sum_{r=0}^{10} \frac{11}{r+1}\,{}^{10}C_r\, 2^{r+1}$$

$$= \frac{1}{11}\sum_{r=0}^{10} {}^{11}C_{r+1} \cdot 2^{r+1}$$

$$= \frac{1}{11}({}^{11}C_1\, 2^1 + \cdots + {}^{11}C_{11} \cdot 2^{11})$$

$$= \frac{1}{11}({}^{11}C_0 \cdot 2^0 + {}^{11}C_1 2^1 + \cdots + {}^{11}C_{11} \cdot 2^{11} - {}^{11}C_0 \cdot 2^0)$$

$$= \frac{1}{11}[(1+2)^{11} - 1] = \frac{3^{11}-1}{11}$$

191 **(a)**

Sum of coefficients of the expansion $\left(\frac{1}{x} + 2x\right)^n$

$$= 6561$$

$\therefore (1+2)^n = 3^8 \Rightarrow 3^n = 3^8 \Rightarrow n = 8$

Now, $T_{r+1} = {}^8C_r 2^{8-r} x^{-8+2r}$

Since, this term is independent of x, then

$$-8 + 2r = 0 \Rightarrow r = 4$$

$\therefore$ Coefficient of independent term, $T_5 = {}^8C_4 \cdot 2^4 = 16 \cdot {}^8C_4$

192 **(d)**

Sum of coefficient of odd powers of x in $(1+x)^{30}$

$= C_1 + C_3 + \ldots + C_{29} = 2^{30-1} = 2^{29}$

193 **(b)**

6th term in the expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$ is

$$T_6 = {}^{10}C_5(2x^2)^5\left(-\frac{1}{3x^2}\right)^5$$

$$= -\frac{10!}{5!\,5!} \times 32 \times \frac{1}{243}$$

$$= -\frac{896}{27}$$

194 **(c)**

$${}^{47}C_4 \sum_{r=1}^{5} {}^{52-r}C_3$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$$

$$= {}^{51}C_3 + {}^{51}C_4 + {}^{52}C_4$$

195 **(b)**

We have,

$$(1 - 2x + 3x^2 - 4x^3 + \cdots)^{-n} = \{(1+x)^{-2}\}^{-n} = (1+x)^{2n}$$

$\therefore$ Coefficient of x^n in $(1 - 2x + 3x^2 - 4x^3 + \cdots)^{-n}$

$= \text{Coefficient of } x^n \text{ in } (1+x)^{2n} = {}^{2n}C_n = \frac{(2n)!}{(n!)^2}$

196 **(c)**

We have,

$$(1+x)^n\left(1 + \frac{1}{x}\right)^n$$

$$= (C_0 + C_1 x + \cdots + C_n x^n)\left\{C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \cdots + \frac{C_n}{x^n}\right\}$$

$\therefore$ Term independent of $x = C_0^2 + C_1^2 + C_2^2 + \cdots + C_n^2$

197 **(a)**

We have,

A = Coeff. of x^r in the expansion of $(1+x)^n$ ${}^nC_r$

B = Coeff. of x^{n-r} in the expansion of $(1+x)^n = {}^nC_{n-r}$

$\because {}^nC_r = {}^nC_{n-r} \quad \therefore A = B$

198 **(b)**

We have,

$$x = \frac{\begin{bmatrix} 729 + 6(2)(243) + 15(4)(81) \\ +20(8)(27) + 15(16)(9) + 6(32)(3) + 64 \end{bmatrix}}{1 + 4(4)6(16) + 4(64) + 256}$$

$$= \frac{\begin{bmatrix} {}^6C_0(3)^6 + {}^6C_1 3^5 2 + {}^6C_2 3^4 2^2 \\ + {}^6C_3 3^3\, 2^3 + {}^6C_4 3^2 2^4 + {}^6C_5 3\, 2^5 + {}^6C_6 2^6 \end{bmatrix}}{{}^4C_0 + {}^4C_1 4 + {}^4C_2 4^2 + {}^4C_3 4^3 + {}^4C_4 4^4}$$

$$\Rightarrow x = \frac{(3+2)^6}{(1+4)^4} = \frac{5^6}{5^4}$$

$$\Rightarrow x = 5^2$$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = 5 - \frac{1}{5} = 4.8$

199 **(b)**

Coefficient of pth, $(p+1)$th and $(p+2)$th terms in the expansion $(1+x)^n$ are ${}^nC_{p-1}$, ${}^nC_p$, ${}^nC_{p+1}$ respectively

Since, these are in AP

$\therefore 2\,{}^nC_p = {}^nC_{p-1} + {}^nC_{p+1}$

$$\Rightarrow 2\frac{n!}{(n-p)!\,p!} = \frac{n!}{(n-p+1)!\,(p-1)!} + \frac{n!}{(n-p-1)!\,(p+1)!}$$

$$\Rightarrow \frac{2}{(n-p)!\,p!} = \frac{p}{(n-p+1)(n-p)!\,p!} + \frac{n-p}{(n-p)!\,(p+1)p!}$$

$$\Rightarrow \frac{2}{1} = \frac{p}{(n-p+1)} + \frac{n-p}{p+1}$$

$$\Rightarrow n^2 - n(4p+1) + 4p^2 - 2 = 0$$

200 **(a)**

$(2^{1/2} + 3^{1/5})^{10} = {}^{10}C_0 2^5 + {}^{10}C_1 2^{9/2} \cdot 3^{1/5} + \ldots\ldots + {}^{10}C_{10} \cdot 3^2$

Thus, sum of rational terms of above expansion $= 2^5 + 3^2 = 41$

201 **(b)**

According to given condition, $T_n = {}^nC_3$

and $T_{n+1} - T_n = 21$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow \frac{1}{6}(n+1)(n)(n-1) - \frac{1}{6}n(n-1)(n-2) = 21$

$\Rightarrow \frac{n(n-1)}{6}[(n+1)-(n-2)] = 21$

$\Rightarrow \frac{n(n-1)\cdot 3}{6} = 21$

$\Rightarrow n(n-1) = 42$

$\Rightarrow n = 7$

202 **(b)**

Since, total number of terms $= 59 + 1 = 60$

$\therefore$ Required sum $= \frac{2^{59}}{2} = 2^{58}$

203 **(d)**

Since, $(1-x)^{-n} = 1 + \frac{n}{1!}x + \frac{n(n+1)}{2!}x^2 + \cdots$

On putting $x = \frac{2x}{1+x}$ on both sides, we get

$$\left(1 - \frac{2x}{1+x}\right)^{-n} = 1 + \frac{n}{1!}\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \cdots$$

$$\Rightarrow 1 + \frac{n}{1!}\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \ldots = \left(\frac{1-x}{1+x}\right)^{-n} = \left(\frac{1+x}{1-x}\right)^n$$

204 **(d)**

General term in the expansion of $(1+3x+2x^2)^6$

$$= \sum \frac{6!}{r_1!\,r_2!\,r_3!}(1)^{r_1}(3x)^{r_2}(2x^2)^{r_3}$$

Where $r_1 + r_2 + r_3 = 6$ ....(i)

For coefficient of x^{11}, we have

$r_2 + 2r_3 = 11$ ...(ii)

Now, from Eqs, (i) and (ii), we get

$r_1 = r_3 - 5$

For $r_3 = 5,\ r_1 = 0$

And $r_2 = 1$

$\therefore$ Coefficient of $x^{11} = \frac{6!}{0!\,1!\,5!}(1)^0(3)^1(2)^5$

$= 6 \times 3 \times 2^5 = 18 \times 32 = 576$

205 **(a)**

We have,

$(1-x)^2\left(x + \frac{1}{x}\right)^{10}$

$= (1 - 2x + x^2)\sum_{r=0}^{10} {}^{10}C_r\, x^{10-2r}$

$= \sum_{r=0}^{10} {}^{10}C_r\, x^{10-2r} - 2\sum_{r=0}^{10} {}^{10}C_r\, x^{11-2x} + \sum_{r=0}^{10} {}^{10}C_r\, x^{12-2x}$

Hence, the term independent of x is

${}^{10}C_5 - 2 \times 0 + {}^{10}C_6 = {}^{10}C_5 + {}^{10}C_6 = {}^{11}C_6 = {}^{11}C_5$

206 **(a)**

Sum of the coefficients in the expansion of $(x - 2y + 3z)^n$ is $(1 - 2 + 3)^n = 2^n$

(Put $x = y = z = 1$)

$\therefore 2^n = 128$

$\Rightarrow n = 7$

Therefore, the greatest coefficient in the expansion of $(1+x)^7$ is ${}^7C_3$ or ${}^7C_4$ because both are equal to 35

207 **(b)**

$19^{2005} + 11^{2005} - 9^{2005}$

$= (10+9)^{2005} + (10+1)^{2005} - (9)^{2005}$

$= (9^{2005} + {}^{2005}C_1(9)^{2004} \times 10 + \ldots) + ({}^{2005}C_0 + {}^{2005}C_1 10 + \ldots) - (9)^{2005}$

$= ({}^{2005}C_1 9^{2004} \times 10 +$ multiple of 10$) + (1$
$+$ multiple of 10$)$

$\therefore$ Unit digit $= 1$

208 **(a)**

$$\sum_{r=0}^{n}(-1)^r\, {}^nC_r\left(\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\ldots \text{upto } m \text{ terms}\right)$$

$$=\sum_{r=0}^{n}(-1)^r\, {}^nC_r\cdot\frac{1}{2^r}+\sum_{r=0}^{n}(-1)^r\cdot {}^nC_r\frac{3^r}{2^{2r}}+\sum_{r=0}^{n}(-1)^r\, {}^nC_r\frac{7^r}{2^{3r}}+\ldots$$

$$=\left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{4}\right)^n+\left(1-\frac{7}{8}\right)^n+\ldots \text{upto } m \text{ terms}$$

$$=\frac{1}{2^n}+\frac{1}{2^{2n}}+\frac{1}{2^{3n}}+\cdots \text{upto m terms}$$

$$=\frac{\frac{1}{2^n}\left(1-\left(\frac{1}{2^n}\right)^m\right)}{\left(1-\frac{1}{2^n}\right)}$$

$$=\frac{2^{mn}-1}{2^{mn}(2^n-1)}$$

209 **(c)**

We have,

Coeff. of $(r+2)^{th}$ term in $(1+x)^{2n} =$

Coeff. of $(3r)^{th}$ term

$\Rightarrow {}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$

$\Rightarrow r+1+3r-1=2n \Rightarrow 4r=2n \Rightarrow n=2r$

210 **(b)**

$$\frac{\left(1+\frac{3}{4}x\right)^{-4}(16)^{1/2}\left(1-\frac{3}{16}x\right)^{1/2}}{(8)^{2/3}\left(1+\frac{x}{8}\right)^{2/3}}$$

$$=\left(1+\frac{3x}{4}\right)^{-4}\left(1-\frac{3x}{16}\right)^{\frac{1}{2}}\left(1+\frac{x}{8}\right)^{-\frac{2}{3}}$$

$$=\left(1+(-4)\frac{3}{4}x\right)\left(1-\left(\frac{1}{2}\right)\frac{3x}{16}\right)\left(1+\left(-\frac{2}{3}\right)\frac{x}{8}\right)$$

$$=(1-3x)\left(1-\frac{3}{32}x\right)\left(1-\frac{x}{12}\right)$$

$= 1-\frac{305}{96}x$ (On neglecting x^2 and higher powers of x)

211 **(c)**

We have,

$(1+x^2)^5(1+x)^4$

$= ({}^5C_0 + {}^5C_1x^2 + {}^5C_2\,x^4 + {}^5C_3\,x^6 + \cdots) \times ({}^4C_0 + {}^4C_1\,x + {}^4C_2\,x^2 + {}^4C_3\,x^3 + {}^4C_4\,x^4$

$\therefore$ Coefficient of $x^5 = {}^5C_1 \times {}^4C_3 + {}^5C_2 \times {}^4C_1$
$= 20 + 40 = 60$

212 **(a)**

We have,

$(1+5\sqrt{2}\,x)^9 + (1-5\sqrt{2}\,x)^9$

$= 2\left\{{}^9C_0 + {}^9C_2(5\sqrt{2}\,x)^2 + \cdots + {}^9C_8(5\sqrt{2}\,x)^8\right\}$

Clearly, it has 5 terms

214 **(d)**

$(1+2x+3x^2+\ldots)^{1/2} = [(1-x)^{-2}]^{1/2}$
$= (1-x)^{-1}$
$= 1+x+x^2+\cdots+x^n+\cdots\infty$

$\therefore$The coefficient of $x^n = 1$

215 **(d)**

Coefficient of $\lambda^n\mu^n$ in $(1+\lambda)^n(1+\mu)^n(\lambda+\mu)^n$

=coefficient of $\lambda^n\mu^n$in

$$\sum_{r=0}^{n}{}^nC_r\lambda^r\sum_{s=0}^{n}{}^nC_s\mu^s\sum_{t=0}^{n}{}^nC_t\lambda^{n-t}\mu^t$$

$$=({}^nC_0)^3+({}^nC_1)^3+({}^nC_2)^3+\ldots=\sum_{r=0}^{n}({}^nC_r)^3$$

216 **(c)**

We have,

$(1+x-2\,x^2)^6$
$= 1 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + 9 \ldots + C_{12}\,x^{12}$

Putting $x = 1, -1$, we get

$0 = 1 + C_1 + C_2 + C_3 + C_4 + \cdots + C_{12}$ ... (i)

$64 = 1 - C_1 + C_2 - C_3 + C_4 - \cdots + C_{12}$ ... (ii)

Adding (i) and (ii), we get

$64 = 2(1 + C_2 + C_4 + \cdots + C_{12})$

$\Rightarrow C_2 + C_4 + \cdots + C_{12} = 31$

217 **(a)**

We have,

Coeff. Of x in $(1+ax)^n = 8$ and, Coeff. Of x^2 in $(1+ax)^n = 24$

$\Rightarrow {}^nC_1a = 8$ and ${}^nC_2a^2 = 24$

$\Rightarrow na = 8$ and $n(n-1)a^2 = 48$

$\Rightarrow 64 - 8a = 48 \Rightarrow a = 2$

$\therefore na = 8 \Rightarrow n = 4$

218 **(a)**

Since, $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_0 + {}^{2n}C_1x + {}^{2n}C_2x^2$
$+\ldots.+ {}^{2n}C_nx^n + \ldots {}^{2n}C_{2n}x^{2n}$

Total number of terms in the expansion $= 2n+1$

$\therefore (n+1)$th term is middle term. This term has greatest coefficient.

Hence, required greatest coefficient = ${}^{2n}C_n$

219 **(a)**

The general term in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

$T_{r+1} = (-1)^r\, {}^{10}C_r \left(\frac{x}{2}\right)^{10-r}\left(\frac{3}{x^2}\right)^r$

$= (-1)^r\, {}^{10}C_r . \frac{3^r}{2^{10-r}} . x^{10-3r}$

For coefficient of x^4 , we have to take $10-3r=4$

$\Rightarrow 3r = 6 \Rightarrow r = 2$

$\therefore$ Coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$

$= (-1)^2 . {}^{10}C_2 . \frac{3^2}{2^8} = \frac{9\times 45}{256} = \frac{405}{256}$

220 **(b)**

Clearly, ${}^nC_r$ is the greatest and n is odd

$\therefore r = \frac{n+1}{2}$ or, $\frac{n-1}{2}$

221 **(a)**

We have, $R = [R] + F$

Let $G = (5\sqrt{5}-11)^{2n+1}$. Then, $0 < G < 1$ as $0 < 5\sqrt{5} - 11 < 1$

Now,

$R - G = (5\sqrt{5}+11)^{2n+1} - (5\sqrt{5}-11)^{2n+1}$

$\Rightarrow R - G = 2\{{}^{2n+1}C_1 (5\sqrt{5})^{2n} (11)^1 + {}^{2n+1}C_3 (5\sqrt{5})^{2n-2} (11)^3 + \cdots + {}^{2n+1}C_{2n+1} (11)^{2n+1}\}$

$\Rightarrow R - G =$ an even integer

$\Rightarrow [R] + F - G =$ an even integer

$\Rightarrow F - G$ is an integer

$\Rightarrow F - G = 0$

$\Rightarrow F = G$

$\Rightarrow RF = RG = (5\sqrt{5}+11)^{2n+1}(5\sqrt{5}-11)^{2n+1} = 4^{2n+1}$

222 **(c)**

Coefficients of $T_5 = {}^nC_4, T_6 = {}^nC_5$ and $T_7 = {}^nC_6$

According to the given condition,

$2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow 2\left[\frac{n!}{(n-5)!\,5!}\right] = \left[\frac{n!}{(n-4)!\,4!} + \frac{n!}{(n-6)!\,6!}\right]$

$\Rightarrow 2\left[\frac{6}{(n-5)}\right] = \left[\frac{5.6}{(n-4)(n-5)} + 1\right]$

$\Rightarrow \frac{12}{(n-5)} = \frac{30 + n^2 - 9n + 20}{(n-4)(n-5)}$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow (n-7)(n-14) = 0$

$\Rightarrow n = 7$ or 14

223 **(a)**

Given that, $R = (2+\sqrt{3})^{2n}$ and $f = R - [R]$

As $0 < 2 - \sqrt{3} < 1$, we get $0 < F = (2-\sqrt{3})^{2n} < 1$

We have, $R + F = (2+\sqrt{3})^{2n} + (2-\sqrt{3})^{2n}$

$= 2\left[{}^{2n}C_0 2^{2n} + {}^{2n}C_2 2^{2n-2}(\sqrt{3})^2 + {}^{2n}C_4 (2^{2n-4})(\sqrt{3})^4 + \ldots + {}^{2n}C_{2n}(\sqrt{3})^{2n}\right]$

$\Rightarrow R + F$ is an even integer

$\Rightarrow [R] + f + F$ is an even integer

$\Rightarrow f + F$ is an integer

But, $0 \le f < 1$ and $0 < F < 1$

$\Rightarrow 0 < f + F < 2$

But the only integer between 0 and 2 is 1. Thus,

$f + F = 1 \Rightarrow 1 - f = F$

Now, $R(1-f) = RF = (2+\sqrt{3})^{2n}(2-\sqrt{3})^{2n}$

$= (4-3)^{2n} = 1^{2n} = 1$

224 **(d)**

$({}^7C_0 + {}^7C_1) + ({}^7C_1 + {}^7C_2) + \cdots + ({}^7C_6 + {}^7C_7)$

$= {}^8C_1 + {}^8C_2 + \cdots + {}^8C_7 + ({}^8C_0 + {}^8C_8) - ({}^8C_0 + {}^8C_8)$

$= 2^8 - 2$

225 **(a)**

We have, ${}^{4n}C_0 + {}^{4n}C_2 x^2 + {}^{4n}C_4 x^4 + \ldots + {}^{4n}C_{4n} x^{4n}$

$= \frac{1}{2}[(1+x)^{4n} + (1-x)^{4n}]$

On putting $x = 1$ and $x = i$, we get

${}^{4n}C_0 + {}^{4n}C_2 + \ldots + {}^{4n}C_{4n} = \frac{1}{2}[2^{4n}]$ ...(i)

and ${}^{4n}C_0 + {}^{4n}C_2 + \ldots + {}^{4n}C_{4n} = \frac{1}{2}[(1+i)^{4n} + (1-i)^{4n}]$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2[{}^{4n}C_0 + {}^{4n}C_4 + \ldots + {}^{4n}C_{4n}] = 2^{4n-1} + \frac{1}{2}[(1+i)^{4n} + (1-i)^{4n}]$

Now, $(1+i)^{4n} + (1-i)^{4n}$

$= \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{4n} + \left[\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)\right]^{4n}$

$= 2^{2n}(\cos n\pi + i\sin n\pi) + 2^{2n}(\cos n\pi - i\sin n\pi)$

$= 2^{2n+1}\cos n\pi = 2^{2n+1}(-1)^n$

$\therefore 2[{}^{4n}C_0 + {}^{4n}C_4 + \ldots + {}^{4n}C_{4n}] = 2^{4n-1} + \frac{1}{2} 2^{2n+1}(-1)^n$

$\Rightarrow {}^{4n}C_0 + {}^{4n}C_4 + \ldots + {}^{4n}C_{4n} = 2^{4n-1} + (-1)^n 2^{2n-1}$

226 **(a)**

Suppose $(s+1)^{\text{th}}$ term contains x^{2r}

We have,

$T_{s+1} = {}^{n-3}C_s\, x^{n-3-s}\left(\frac{1}{x^2}\right)^s = {}^{n-3}C_s\, x^{n-3-3s}$

This will contain x^{2r}, if

$n-3-3s=2r$

$\Rightarrow s=\frac{n-3-2r}{3}$

$\Rightarrow s=\frac{n-2r}{3}-1$

$\Rightarrow s+1=\frac{n-2r}{3}$

$\Rightarrow n-2r=3(s+1)$

$\Rightarrow n-2r$ is a positive integral multiple of 3

228 **(d)**

Let $1+\frac{1}{3}x+\frac{1\cdot4}{3\cdot6}x^2+\frac{1\cdot4\cdot7}{3\cdot6\cdot9}x^3+\ldots=(1+y)^n$

$=1+ny+\frac{n(n-1)}{2!}y^2+\ldots$

On comparing the terms, we get

$ny=\frac{1}{3}x, \frac{n(n-1)}{2!}y^2=\frac{1\cdot4}{3\cdot6}x^2$

On solving, we get

$n=-\frac{1}{3}, \quad y=-x$

$\therefore$ Required expansion is $(1-x)^{-1/3}$

229 **(c)**

On putting $x=1$, we get the sum of coefficient of $(x^2-x-1)^{99}$

$=(1-1-1)^{99}=(-1)^{99}=-1$

231 **(b)**

${}^{15}C_8+{}^{15}C_9-{}^{15}C_6-{}^{15}C_7$

$={}^{15}C_8+{}^{15}C_9-{}^{15}C_9-{}^{15}C_8$

$=0$

232 **(a)**

$(1-x+x^2)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$

On putting $x=1$, we get

$(1-1+1)^n=a_0+a_1+a_2+\ldots+a_{2n}$

$\Rightarrow 1=a_0+a_1+a_2+\ldots+a_{2n}$ ...(i)

Again, putting $x=-1$, we get

$3^n=a_0-a_1+a_2-\ldots+a_{2n}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$\frac{3^2+1}{2}=a_0+a_2+a_4+\ldots+a_{2n}$

233 **(c)**

Let us take

$a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}=(1+x+x^2)^n$

On differentiating both sides w.r.t. x, we get

$a_1+2a_2x+\ldots+2na_{2n}x^{2n-1}$

$=n(1+x+x^2)^{n-1}(2x+1)$

Put $x=-1$

$\Rightarrow a_1-2a_2+3a_3-\ldots-2na_{2n}=-n$

234 **(d)**

$\because (k^2-3)=\frac{{}^{n-1}C_r}{{}^{n-1}C_{r+1}}=\frac{{}^{n-1}C_r}{\left(\frac{n}{r+1}\right){}^{n-1}C_r}=\left(\frac{r+1}{n}\right)$ ...(i)

$\because 0\le r\le n-1$

$\Rightarrow 1\le r+1\le n$

$\Rightarrow \frac{1}{n}\le\frac{r+1}{n}\le 1$

$\Rightarrow \frac{1}{n}\le k^2-3\le 1$ [from Eq. (i)]

$\Rightarrow 3+\frac{1}{n}\le k^2\le 4$

When $n\to\infty, 3\le k^2\le 4$

or $k\in[-2,-\sqrt{3}]\cup(\sqrt{3},2]$

235 **(d)**

The general term in the expansion of $\left(x\cos\alpha+\frac{\sin\alpha}{x}\right)^{20}$ is ${}^{20}C_r(x\cos\alpha)^{20-r}\left(\frac{\sin\alpha}{x}\right)^r=$

${}^{20}C_r x^{20-2r}(\cos\alpha)^{20-r}(\sin\alpha)^r$

For this term to be independent of x, we get

$20-2r=0\Rightarrow r=10$

Let $\beta=$Term independent of x

$={}^{20}C_{10}(\cos\alpha)^{10}(\sin\alpha)^{10}$

$={}^{20}C_{10}(\cos\alpha\sin\alpha)^{10}$

$={}^{20}C_{10}\left(\frac{\sin 2a}{2}\right)^{10}$

Thus, the greatest possible value of β is

${}^{20}C_{10}\left(\frac{1}{2}\right)^{10}$

236 **(c)**

Since $(n+2)^{th}$ term is the middle term in the expansion of $(1+x)^{2n+2}$. Therefore,

$p={}^{2n+2}C_{n+1}$

Since $(n+1)^{th}$ and $(n+2)^{th}$ terms are two middle terms in the expansion of $(1+x)^{2n+1}$. Therefore,

$q={}^{2n+1}C_n$ and $r={}^{2n+1}C_{n+1}$

But, ${}^{2n+1}C_n+{}^{2n+1}C_{n+1}={}^{2n+2}C_{n+1}$

$\Rightarrow q+r=p$

237 **(a)**

Given

${}^mC_0+{}^mC_1+{}^mC_2=46$

$\Rightarrow 2m+m(m-1)=90$

$\Rightarrow m^2+m-90=0\Rightarrow m=9$ as $m>0$

Now, $(r+1)$th term of $\left(x^2+\frac{1}{x}\right)^m$ is

${}^mC_r(x^2)^{m-r}\left(\frac{1}{x}\right)^r={}^mC_r x^{2m-3r}$

For this to be independent of x put

$2m-3r=0\Rightarrow r=6$

$\therefore$ Coefficient of the term independent of x is

${}^9C_6 = 84$

238 **(a)**

Since ${}^nC_{r-1}$, ${}^nC_r$ and ${}^nC_{r+1}$ are in A. P.

$\therefore 2\,{}^nC_r = {}^nC_{r-1} + {}^nC_{r+1}$

$$\Rightarrow 2\frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-r+1)!\,(r-1)!} + \frac{n!}{(n-r-1)!\,(r+1)!}$$

$\Rightarrow n^2 - n(4r+1) + 4r^2 - 2 = 0$

$\Rightarrow n$ is a root of the equation $x^2 - x(4r+1) \pm 4r^2 - 2 = 0$

239 **(a)**

$$(1+x+x^2)^{-3} = \left[\frac{1}{(1+x+x^2)}\right]^3 = \left[\frac{1-x}{1-x^3}\right]^3 = (1-x)^3(1-x^3)^{-3}$$

$$= (1-x^3-3x^2+3x)(1+3x^3+6x^6+\cdots)$$

$\therefore$ Coefficient of x^6 in $(1+x+x^2)^{-3}$

$= 6 - 3 = 3$

240 **(d)**

$$\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$$

$$= \left(1+\frac{n}{1}\right)\left(1+\frac{n(n-1)}{2n}\right)\ldots\left(1+\frac{1}{n}\right)$$

$$= \left(\frac{1+n}{1}\right)\left(\frac{1+n}{2}\right)\ldots\left(\frac{1+n}{n}\right) = \frac{(1+n)^n}{n!}$$

241 **(a)**

$(1+3x+3x^2+x^3)^{20} = (1+x)^{60}$

$\therefore$ Coefficent of x^{20}in $(1+x)^{60}$is ${}^{60}C_{20}$or ${}^{60}C_{40}$.

242 **(c)**

The general term in the expansion of $(x\sin\alpha + x^{-1}\cos\alpha)^{10}$ is

$T_{r+1} = {}^{10}C_r(x\sin\alpha)^{10-r}(x^{-1}\cos\alpha)^r$

$= {}^{10}C_r(\sin\alpha)^{10-r}(\cos\alpha)^r x^{10-2r}$

For the term independent of x, put

$10 - 2r = 0 \Rightarrow r = 5$

$\therefore$ Coefficient of term independent of x, is

$${}^{10}C_5(\sin\alpha)^5(\cos\alpha)^5 = {}^{10}C_5\left(\frac{1}{2^5}\right)(\sin 2\alpha)^5$$

$$\le \frac{1}{2^5}({}^{10}C_5) \quad [\because \sin(2\alpha) \le 1]$$

243 **(b)**

The general term in the expansion of $(1+2x+3x^2)^{10}$ is

$$\frac{10!}{r!\,s!\,t!}1^r(2x)^s(3x^2)^t, \text{where } r+s+t = 10$$

$$= \frac{10!}{r!\,s!\,t!}2^s \times 3^t \times x^{s+2t}$$

We have to find a_1 i.e. the coefficient of x

For the coefficient of x^1, we must have

$s + 2t = 1$

But, $r + s + t = 10$

$\therefore s = 1 - 2t$ and $r = 9 + t$, where $0 \le r, s, t \le 10$

Now,$t = 0 \Rightarrow s = 1, r = 9$

For other values of t, we get negative values of s.

So, there is only one term containing x and its coefficient is

$$\frac{10!}{9!\,1!\,0!} \times 2^1 \times 3^0 = 20$$

Hence, $a_1 = 20$

<u>ALITER</u> we have,

$(1+2x+3x^2)^{10}$

$$= {}^{10}C_0 + {}^{10}C_1(2x+3x^2) + {}^{10}C_2(2x+3x^2)^2 + \cdots + {}^{10}C_0(2x+3x^2)^{10}$$

$\therefore a_1 =$ Coeff. of $x = 20$

244 **(b)**

Since, the coefficient of given terms are ${}^mC_{r-1}$, ${}^mC_r$, ${}^mC_{r+1}$ respectively and they are in AP.

$$\therefore \quad {}^mC_{r-1} + {}^mC_{r+1} = 2\,{}^mC_r$$

$$\Rightarrow \frac{m!}{(r-1)!\,(m-r+1)!} + \frac{m!}{(r+1)!\,(m-r-1)!} = 2\frac{m!}{r!\,(m-r)!}$$

$$\Rightarrow \frac{1}{(m-r+1)(m-r)} + \frac{1}{(r+1)r} = \frac{2}{r!\,(m-r)}$$

$$\Rightarrow \frac{r(r+1)+(m-r+1)(m-r)}{r(r+1)(m-r+1)(m-r)} = \frac{2}{r(m-r)}$$

$\Rightarrow r^2 + r + m^2 + r^2 - 2mr + m - r$

$= 2(mr - r^2 + r + m - r + 1)$

$\Rightarrow 4r^2 - 4mr - m - 2 + m^2 = 0$

$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$

245 **(b)**

General term, $T_{r+1} = {}^6C_r x^{6-r}\left(\frac{1}{x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^6C_r x^{6-3r}$

For term independent of x, put

$6 - 3r = 0$

$\Rightarrow r = 2$

$\therefore$ Coefficient of independent term $= {}^6C_2 = 15$

246 **(a)**

We have,

$$T_{n+1} = \left(\frac{1}{3 \times \sqrt[3]{9}}\right)^{\log_3 8}$$

$$\Rightarrow {}^nC_n\left(\sqrt[3]{2}\right)^0\left(-\frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3 \times \sqrt[3]{9}}\right)^{\log_3 8}$$

$\Rightarrow \left(-\frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3^{5/3}}\right)^{\log_3 8}$

$\Rightarrow (-1)^n 2^{-n/2} = \left(3^{-5/3}\right)^{\log_3 2^3}$

$\Rightarrow (-1)^n 2^{-n/2} = (3^{\log_3 2^{-5}})$

$\Rightarrow (-1)^n 2^{-n/2} = 2^{-5} \Rightarrow \frac{n}{2} = 5 \Rightarrow n = 10$

247 **(b)**

Given, $a_0 = 1, a_{n+1} = 3n^2 + n + a_n$

$\Longrightarrow a_1 = 3(0) + 0 + a_0 = 1$

$\Longrightarrow a_2 = 3(1)^2 + 1 + a_1 = 3 + 1 + 1 = 5$

From option (b),

Let $P(n) = n^3 - n^2 + 1$

$\therefore \quad P(0) = 0 - 0 + 1 = 1 = a_0$

$P(1) = 1^3 - 1^2 + 1 = 1 = a_1$

and $P(2) = (2)^3 - (2)^2 + 1 = 5 = a_2$

248 **(b)**

General term , $T_{r+1} = {}^{10}C_r (x^2)^{10-r} \left(-\frac{1}{x^3}\right)^r$

$= {}^{10}C_r x^{20-5r}(-1)^r$

Since, this term condition x^{-10}

$\therefore \quad 20 - 5r = -10 \Longrightarrow r = 6$

$\therefore$ Coefficient of $x^{-10} = {}^{10}C_6(-1)^6 = 210$

251 **(c)**

The sum of the coefficients of the polynomial $(a^2x^2 - 2ax + 1)^{51}$ is obtained by putting $x = 1$

Therefore, by given condition $(a^2 - 2a + 1)^{51} = 0$

$\Rightarrow a = 1$

252 **(b)**

Let $S = 1 + \frac{2}{4} + \frac{2 \cdot 5}{4 \cdot 8} + \frac{2 \cdot 5 \cdot 8}{4 \cdot 8 \cdot 12} + \cdots$

On comparing with

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$

we get $\quad nx = \frac{2}{4} \quad$ .... (i)

and $\quad \frac{n(n-1)}{2!}x^2 = \frac{2 \cdot 5}{4 \cdot 8} \quad$ ... (ii)

From Eqs, (i) and (ii)

$$\frac{\frac{n(n-1)}{2!}x^2}{x^2x^2} = \frac{\frac{2 \cdot 5}{4 \cdot 8}}{\frac{2 \cdot 2}{4 \cdot 4}}$$

$\Longrightarrow \frac{n-1}{n} = \frac{5}{2} \Longrightarrow n = -\frac{2}{3}$

On putting the value of n in Eq. (i) we get

$-\frac{2}{3}x = \frac{2}{4} \Longrightarrow x = -\frac{3}{4}$

$\therefore S = (1 + x)^n = \left(1 - \frac{3}{4}\right)^{-2/3} = \left(\frac{1}{4}\right)^{-2/3} = \sqrt[3]{16}$

253 **(b)**

We have,

$(x + 3)^{n-1} + (x + 3)^{n-2}(x + 2) + \cdots + (x + 2)^{n-1}$

$= \frac{(x + 3)^n - (x + 2)^n}{(x + 3) - (x + 2)} = (x + 3)^n - (x + 2)^n$

$\therefore$ Coefficient of x^r in the given expression

$=$ Coeff. of x^r in $\{(x + 3)^n - (x + 2)^n\}$

$= {}^nC_r\, 3^{n-r} - {}^nC_r\, 2^{n-r} = {}^nC_r(3^{n-r} - 2^{n-r})$

254 **(c)**

Let $S = C_1 + 2C_2 + 3C_3 + \cdots + nC_n = \sum_{r=1}^{n} r \cdot {}^nC_r$

$= \sum_{r=1}^{n} r \cdot \frac{n}{r}\, {}^{n-1}C_{r-1} \quad \left[\because {}^nC_r = \frac{n}{r}\, {}^{n-1}C_{r-1}\right]$

$= n \sum_{r=1}^{n} {}^{n-1}C_{r-1}$

$= n[{}^{n-1}C_0 + {}^{n-1}C_1 + {}^{n-1}C_2 + \cdots + {}^{n-1}C_{n-1}]$

$= n2^{n-1}$

255 **(b)**

Given expression $\frac{\sqrt{1+x} + \sqrt[3]{(1-x)^2}}{1+x+\sqrt{1+x}}$ can be rewritten as

$\frac{(1 + x)^{1/2} + (1 - x)^{2/3}}{1 + x + (1 + x)^{1/2}}$

$= \frac{\left[1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots\right] + \left[1 - \frac{2}{3}x - \frac{1}{9}x^2 - \ldots\right]}{1 + x + \left[1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots\right]}$

$= \frac{2 - \frac{1}{6}x - \frac{17}{72}x^2 + \ldots}{2 + \frac{3}{2}x - \frac{1}{8}x^2 + \ldots} = \frac{\left[1 - \frac{1}{12}x - \frac{17}{144}x^2 + \ldots\right]}{\left[1 + \frac{3}{4}x - \frac{1}{16}x^2 + \ldots\right]}$

$= \left[1 - \frac{1}{12}x - \frac{17}{144}x^2 + \ldots\right]\left[1 + \frac{3}{4}x - \frac{1}{16}x^2 + \ldots\right]^{-1}$

$= 1 - \frac{5}{6}x + \ldots$

$= 1 - \frac{5}{6}x$ (On neglecting the higher powers of x)

256 **(b)**

The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$

$=$ coefficient of x^n in $(1 - x)^n$

$+$ The coefficient of x^{n-1} in $(1 - x)^n$

$= (-1)^n \frac{n!}{n!\,0!} + (-1)^{n-1} \frac{n!}{1!\,(n-1)!}$

$= (-1)^n(1 - n)$

257 **(b)**

Middle term of $(x - a)^8$is

$T_5 = {}^8C_4 x^4 (-a)^4 = {}^8C_4 x^4 a^4$

258 **(c)**

We have,

Coeff. of r^{th} term in $(1+x)^{20}$ = Coeff. of $(r+4)^{th}$ term in $(1+x)^{20}$

$\Rightarrow {}^{20}C_{r-1} = {}^{20}C_{r+3}$

$\Rightarrow (r-1)+(r+3) = 20 \Rightarrow 2r+2 = 20 \Rightarrow r = 9$

259 **(c)**

Putting $x = 1$ and $x = -1$ in the given expansion and adding, we get

$2[1 + a_2 + a_4 + \dots + a_{12}] = (-2)^6$

$\Rightarrow a_2 + a_4 + \dots + a_{12} = 31$

260 **(d)**

We have,

$$\frac{1}{81^n} - \frac{10}{81^n}\,{}^{2n}C_1 + \frac{10^2}{81^n}\,{}^{2n}C_2 - \frac{10^3}{81^n}\,{}^{2n}C_3 + \dots + \frac{10^{2n}}{81^n}$$

$$= \frac{1}{81^n}\{{}^{2n}C_0 - {}^{2n}C_1\,10^1 + {}^{2n}C_2\,10^2 - {}^{2n}C_3\,10^3 + \dots + {}^{2n}C_{2n}10^{2n}\}$$

$$= \frac{1}{81^n}(1-10)^{2n} = \frac{(-9)^{2n}}{81^n} = \frac{81^n}{81^n} = 1$$

261 **(a)**

$$\left(\frac{{}^{50}C_0}{1} + \frac{{}^{50}C_2}{3} + \frac{{}^{50}C_4}{5} + \dots + \frac{{}^{50}C_{50}}{51}\right)$$

$$= \frac{1}{1} + \frac{50\times 49}{3\times 2!\times} + \frac{50\times 49\times 48\times 47}{5\times 4!} + \dots$$

$$= \frac{1}{51}\left(51 + \frac{51\times 50\times 49}{3!} + \frac{51\times 50\times 49\times 48\times 47}{5!} + \dots\right)$$

$$= \frac{1}{51}({}^{51}C_1 + {}^{51}C_3 + {}^{51}C_5 + \dots) = \frac{1}{51}\cdot 2^{51-1}$$

$$= \frac{2^{50}}{51}$$

262 **(a)**

As we have $A^2 = 2A - I$

$\Longrightarrow\ A^2A = (2A-I)A = 2A^2 - IA$

$\Longrightarrow\ A^3 = 2(2A-I) - IA = 3A - 2I$

Similarly, $A^4 = 4A - 3I$

$A^5 = 5A - AI$

$A^n = 5A - 4I$

… … … … … …

… … … … … …

$A^n = nA - (n-1)I$

263 **(a)**

We have,

$$\sum_{r=0}^{2n} a_r(x-100)^r = \sum_{r=0}^{2n} b_r(x-101)^r$$

$$\Rightarrow \sum_{r=0}^{2n} b_r\, t^r = \sum_{r=0}^{2n} a_r\,(1+t)^r,\ \text{where } t = x - 101$$

On equating the coefficients of t^n on both sides, we get

$$b_n = a_n\,{}^nC_n + a_{n+1}\,{}^{n+1}C_n + a_{n+2}\,{}^{n+2}C_n + \dots + a_{2n}\,{}^{2n}C_n$$

$$\Rightarrow b_n = \sum_{r=n}^{2n} a_r\,{}^rC_n$$

$$= \sum_{r=n}^{2n} 2^r$$

$$= 2^n \sum_{r=n}^{2n} 2^{r-n} = 2^n(2^{n+1}-1)$$

264 **(b)**

We have,

$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \dots + (x+2)^{n-1}$$

$$= \frac{(x+3)^n - (x+2)^n}{(x+3)-(x+2)} = (x+3)^n - (x+2)^n$$

$$\left(\because \frac{x^n - a^n}{x-a} = x^{n-1} + x^{n-2}a^1 + x^{n-3}a^2 + \dots + a^{n-1}\right)$$

Therefore, the coefficient of x^r in the given expression

=coefficient of x^r in $[(x+3)^n - (x+2)^n]$

$= {}^nC_r 3^{n-r} - {}^nC_r 2^{n-r}$

$= {}^nC_r(3^{n-r} - 2^{n-r})$

266 **(d)**

The sum of the magnitudes of the coefficients is obtained by replacing x by -1 in $(1 - x + x^2 - x^3)^n$

Hence, required sum $= (1+1+1+1)^n = 4^n$

267 **(c)**

Let x^7 occur in $(r+1)^{\text{th}}$ term

Now,

$$T_{r+1} = \frac{x^{-3}(-3)(-3-1)(-3-2)\dots(-3-r+1)}{r\,!}\left(-\frac{2}{\ }\right.$$

$$\Rightarrow T_{r+1} = \frac{3\cdot 4\cdot 5\dots(r+2)}{r\,!}2^r\,x^{r-3}$$

This will contain x^7, if

$\therefore r - 3 = 7 \Rightarrow r = 10$

$$\therefore \text{Coefficient of } x^7 = \frac{3\cdot 4\cdot 5\dots(10+2)}{10\,!}\cdot 2^{10}$$

$$= 66\times 2^{10} = 67584$$

268 **(d)**

The given expansion

$$= \sum_{r=0}^{n} (x-r)(y-r)(z-r)(-1)^r C_r$$

$$= \sum_{r=0}^{n} (-1)^r xyz\, C_r \sum_{r=0}^{n} (-1)^r r(x+y+z) C_r$$

$$+ \sum_{r=0}^{n} (-1)^r r^2 (xy+yz+zx) C_r$$

$$- \sum_{r=0}^{n} (-1)^r xyz\, r^3 C_r$$

$$= xyz \sum_{r=0}^{n} (-1)^r C_r - (x+y+z) \sum_{r=0}^{n} (-1)^r r\, C_r$$

$$+ (xy+yz+zx) \sum_{r=0}^{n} (-1)^r r^2 C_r$$

$$- xyz \sum_{r=0}^{n} (-1)^r r^3 C_r$$

$$= xyz \times 0 - (x+y+z) \times 0 + (xy+yz+zx) \times 0 - xyz \times 0 = 0$$

269 **(b)**

We know that,

$$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + n\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$$

On putting $n = 15$, then $\frac{15 \times (15+1)}{2} = 15 \times 8 = 120$

270 **(b)**

We have,

$$17^{1995} + 11^{1995} - 7^{1995}$$

$$= (7+10)^{1995} + (1+10)^{1995} - 7^{1995}$$

$$= \{7^{1995} + {}^{1995}C_1 7^{1994} \cdot 10^1 + {}^{1995}C_2 \cdot 7^{1993} 10^2 + \cdots + {}^{1995}C_{1995} \cdot 10^{1995}\}$$
$$+ \{{}^{1995}C_0 + {}^{1995}C_1\, 10^1 + {}^{1995}C_2\, 10^2 + \cdots + {}^{1995}C_{1995} 10^{1995}\} - 7^{1995}$$

$$= \{{}^{1995}C_1 7^{1994} \cdot 10^1 + \cdots + 10^{1995}\} + \{{}^{1995}C_1 10^1 + \cdots + {}^{1995}C_{1995} 10^{1995}\} + 1$$

= (a multiple of 10) +1

Thus, the unit's digit is 1

271 **(a)**

$${}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3$$

$$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$$

$$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$= {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3 = {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$$

272 **(b)**

$$\left(x - \frac{1}{x}\right)^4 \left(x + \frac{1}{x}\right)^3$$

$$= \left({}^4C_0 x^4 - {}^4C_1 x^2 + {}^4C_2 - {}^4C_3 \frac{1}{x^2} + {}^4C_4 \frac{1}{x^4}\right) \times \left({}^3C_0 x^3 + {}^3C_1 x + {}^3C_2 \frac{1}{x} + {}^3C_3 \frac{1}{x^3}\right)$$

Clearly, there is no term from x on RHS, therefore the term independent of x on LHS is zero.

273 **(b)**

Coefficient of x^2y^2 in $(x+y+z+t)^4 = \frac{4!}{2!2!} = 6$

and

coefficient of yzt^2 in $(x+y+z+t)^4$

$$= \frac{4!}{1!\,1!\,1!\,2!} = 12$$

Also, coefficients of $xyzt$ in

$$(x+y+z+t)^4 = \frac{4!}{1!\,1!\,1!\,1!} = 24$$

$\therefore$ Required ratio is $6:12:24 = 1:2:4$

274 **(b)**

The general term in the expansion of $(1+2x+3x^2)^{10}$

is $\sum \frac{10!}{r!\,s!\,t!} 1^r (2x)^5 (3x^2)^t$

$$= \frac{10!}{r!\,s!\,t!} 2^s \times 3^t \times x^{s+2t}$$

Where $r + s + t = 10$

We have to find a_1 *ie*, coefficient of x

For the coefficient of x^1, we must have

$$s + 2t = 1$$

But $\quad r + s + t = 10$

$\therefore s = 1 - 2t$ and $r = 9 + t$

Where $0 \le r, s, t \le 10$

Now, $t = 0 \Rightarrow s = 1, r = 9$

For other, values of t, we get negative value s. So, there is only one term containing x and its coefficient is

$$\frac{10!}{9!\,1!\,0!} 2^1 \times 3^0 = 20$$

Hence, $a_1 = 20$

Alternate On differentiating given equation w. r. t. x, we get

$$10(1+2x+3x^2)^9 = a_1 + 2a_2 x + \ldots + 20a_{20}x^{19}$$

Put $x = 0$, we get

$$20 = a_1$$

275 **(b)**

$$(1+x^2)^5(1+x)^4$$

$$= ({}^5C_0 + {}^5C_1 x^2 + {}^5C_2 x^4 + \cdots)({}^4C_0 + {}^4C_1 x + {}^4C_2 x^2 + {}^4C_3 x^3 + {}^4C_4 x^4)$$

The coefficient of x^5 in $[1+x^2)^5(1+x)^4]$

$$= {}^5C_2 \cdot {}^4C_1 + {}^5C_1 \cdot {}^4C_3$$

$$= 10 \cdot 4 + 4 \cdot 5 = 60$$

276 **(c)**

We have,

$$^{20}C_0+{}^{20}C_1+{}^{20}C_2+{}^{20}C_3+\cdots+{}^{20}C_{10}+{}^{20}C_{11}+\cdots+{}^{20}C_{20}=2^{20}$$

$$\Rightarrow\{{}^{20}C_0+{}^{20}C_{20}\}+\{{}^{20}C_1+{}^{20}C_{19}\}+\cdots+\{{}^{20}C_9+{}^{20}C_{11}\}+{}^{20}C_{10}=2^{20}$$

$$\Rightarrow 2\{{}^{20}C_0+{}^{20}C_1+{}^{20}C_2+\cdots+{}^{20}C_9\}+{}^{20}C_{10}=2^{20}$$

$$\Rightarrow 2\{{}^{20}C_0+{}^{20}C_1+\cdots+{}^{20}C_{10}\}=2^{20}+{}^{20}C_{10}$$

$$\Rightarrow {}^{20}C_0+{}^{20}C_1+\cdots+{}^{20}C_{10}=2^{19}+\frac{1}{2}\,{}^{20}C_{10}$$

277 **(a)**

Last term of $\left(2^{1/3}-\frac{1}{\sqrt{2}}\right)^n$ is

$$T_{n+1}={}^nC_n(2^{1/3})^{n-n}\left(-\frac{1}{\sqrt{2}}\right)^n$$

$$={}^nC_n(-1)^n\frac{1}{2^{n/2}}=\frac{(-1)^n}{2^{n/2}}$$

Also, we have

$$\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}=3^{-(5/3)\log_3 2^3}=2^{-5}$$

Thus, $\frac{(-1)^n}{2^{n/2}}=2^{-5}\Rightarrow\frac{(-1)^n}{2^{n/2}}=\frac{(-1)^{10}}{2^5}$

$$\Rightarrow\frac{n}{2}=5\Rightarrow n=10$$

Now, $T_5=T_{4+1}={}^{10}C_4(2^{1/3})^{10-4}\left(-\frac{1}{\sqrt{2}}\right)^4$

$$=\frac{10!}{4!\,6!}(2^{1/3})^6(-1)^4(2^{-1/2})^4$$

$$=210(2)^2(1)(2^{-2})=210$$

278 **(b)**

Let T_{r+1} be the $(r+1)^{th}$ term in the expansion of $\left(x^2-\frac{1}{x^3}\right)^{10}$. Then, $T_{r+1}={}^{10}C_r\,x^{20-5r}(-1)^r$

This will contain x^{-10}, if $20-5r=-10\Rightarrow r=6$

$\therefore$ Coefficient of $x^{-10}={}^{10}C_6(-1)^6={}^{10}C_6=210$

280 **(b)**

$$\frac{C_1}{C_0}+2\cdot\frac{C_2}{C_1}+3\cdot\frac{C_3}{C_2}+\cdots+n\cdot\frac{C_n}{C_{n-1}}=\frac{1}{k}\,n(n+1)$$

$$\Rightarrow\sum_{r=1}^{n}r\cdot\frac{^nC_r}{^nC_{r-1}}=\frac{1}{k}\,n(n+1)$$

$$\Rightarrow\sum_{r=1}^{n}(n-r+1)=\frac{1}{k}n(n+1)$$

$$\Rightarrow n+(n-1)+(n-2)+\cdots+1.=\frac{1}{k}n(n+1)$$

$$\Rightarrow\frac{n(n+1)}{2}=\frac{1}{k}\,n(n+1)$$

$$\Rightarrow k=2$$

281 **(a)**

We have,

$$(1+x)^p+(1+x)^{p+1}+\cdots+(1+x)^n$$

$$=\frac{(1+x)^p\{(1+x)^{n-p+1}-1\}}{(1+x)-1}$$

$$=\frac{1}{x}\{(1+x)^{n+1}-(1+x)^p\}$$

$\therefore$ Coefficient of x^m

= Coefficient of x^{m+1} in $(1+x)^{n+1}$

$$={}^{n+1}C_{m+1}$$

282 **(c)**

Given that,

$(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots+{}^nC_nx^n$

Let $S_n=\frac{^nC_1}{^nC_0}+\frac{2\,^nC_2}{^nC_1}+\frac{3\,^nC_3}{^nC_2}+\ldots+\frac{n\,^nC_n}{^nC_{n-1}}$

Put $n=1,2,3,\ldots$, then

$$S_1=\frac{^1C_1}{^1C_0}=1,$$

$$S_2=\frac{^2C_1}{^2C_0}+2\frac{^2C_2}{^2C_1}$$

$$=\frac{2}{1}+2\cdot\frac{1}{2}=2+1=3$$

By taking option, (put $n=1,2,\ldots$) (a) and (b) does not hold condition, but option (c) satisfies.

283 **(a)**

Let the term containing x^7 in the expansion of

$$\left(ax^2+\frac{1}{bx}\right)^8 \text{ is } T_{r+1}.$$

$$\therefore\quad T_{r+1}={}^8C_r(ax^2)^{8-r}\left(\frac{1}{bx}\right)^r$$

$$={}^8C_r\frac{a^{8-r}}{b^r}x^{16-3r}$$

Since, this term contains x^7.

$\therefore\quad 16-3r=7$

$\Longrightarrow\quad r=3$

$\therefore$ Coefficient of x^7 in the expansion of $\left(ax^2+\frac{1}{bx}\right)^8$

$$={}^8C_3\cdot\frac{a^5}{b^3}$$

Also, the term containing x^{-7} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^8$ is T_{R+1}

$$T_{R+1}={}^8C_R(a\,x)^{8-R}\left(-\frac{1}{bx^2}\right)^R$$

$$=(-1)^R\,{}^8C_R\frac{a^{8-R}}{b^R}x^{8-3R}$$

Since, this term contains x^{-7}

$\therefore\quad 8-3R=-7$

$\Longrightarrow\quad R=5$

$\therefore$ Coefficient of x^{-7} in the expansion of $\left(a x - \frac{1}{bx^2}\right)^8$

$$= (-1)^5\, {}^8C_5 \cdot \frac{a^3}{b^5}$$

According to the given condition,

$$\left|{}^8C_3 \cdot \frac{a^5}{b^3}\right| = \left|{}^8C_5 \cdot \frac{a^3}{b^5}\right|$$

$\Rightarrow \quad a^2b^2 = 1 \Rightarrow ab = 1$

284 **(b)**

We have,

$(x+a)^n = T_0 + T_1 + T_2 + \cdots + T_n \quad$ ... (i)

Replacing a by ai and $-ai$ respectively in (i), we get

$(a+ai)^n = (T_0 - T_2 + T_4 - T_6 + \cdots) + i(T_1 - T_3 + T_5 - \cdots)$ ... (ii)

And,

$(x-ia)^n = (T_0 - T_2 + T_4 - T_6 + \cdots) - i(T_1 - T_3 + T_5 - \cdots)$ ... (iii)

Multiplying (ii) and (iii), we get

$(x+ai)^n(x-ia)^n = (T_0 - T_2 + T_4 - T_6 - \cdots)^2 + (T_1 - T_3 + T_5 - \cdots)^2$

$\Rightarrow (x^2+a^2)^n = (T_0 - T_2 + T_4 - T_6 \ldots)^2 + (T_1 - T_3 + T_5 - \cdots)^2$

285 **(c)**

$$\because \sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i} = \sum_{i=0}^{m} {}^{10}C_i \cdot {}^{20}C_{m-1}$$

= Coefficient of x^m in the expansion of $(1+x)^{10}(1+x)^{20}$

$= {}^{30}C_m$

It is maximum, when

$m = \frac{30}{2} = 15$

286 **(d)**

$a_1 = {}^nC_1, a_2 = {}^nC_2$

$a_3 = {}^nC_3$

$\because a_1, a_2$ and a_3 are in AP

$\Rightarrow \quad 2a_2 = a_1 + a_3$

$\Rightarrow 2 \cdot {}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow 2 \cdot \frac{n(n-1)}{2!} = n + \frac{n(n-1)(n-2)}{3!}$

$\Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow (n-2)(n-7) = 0$

$\Rightarrow n = 7 \qquad (\because n = 2$ is not Possible)

287 **(c)**

$(1+x)^{15} = a_0 + a_1x + a_2x^2 + \ldots + a_{15}x^{15}$

$\Rightarrow {}^{15}C_0 + {}^{15}C_1x + {}^{15}C_2x^2 + \ldots + {}^{15}C_{15}x^{15} = a_0 + a_1x + a_2x^2 + \ldots + a_{15}x^{15}$

Equating the coefficient of various powers of x, we get

$a_0 = {}^{15}C_0, a_1 = {}^{15}C_1, a_2 = {}^{15}C_2, \ldots a_{15} = {}^{15}C_{15}$

$$\therefore \sum_{r=1}^{15} r\frac{a_r}{a_{r-1}} = \sum_{r=1}^{15} r\frac{{}^{15}C_r}{{}^{15}C_{r-1}}$$

$$= \sum_{r=1}^{15} r\frac{\frac{15!}{r!(15-r)!}}{\frac{15}{(r-1)(15-r+1)!}}$$

$$= \sum_{r=1}^{15} \frac{r(r-1)!\,(15-r+1)!}{r!\,(15-r)!}$$

$$= \sum_{r=1}^{15} 15 - r + 1$$

$= 15 + 14 + 13 + \ldots + 2 + 1$

$= \frac{15(15+1)}{2} = 120$

288 **(b)**

Coefficient of x^5 in $(1+x^2)^5(1+x)^4$

= Coefficient of x^5 in $({}^5C_0 + {}^5C_1\,x^2 + {}^5C_2\,x^4 + \cdots)(1+x)^4$

$= {}^5C_1 \times$ Coefficient of x^3 in $(1+x)^4 + {}^5C_2 \times$ Coefficient of x in $(1+x)^4$

$= {}^5C_1 \times {}^4C_3 + {}^5C_2 \times {}^4C_1 = 20 + 40 = 60$

289 **(b)**

Since, $(1-x)^{-2} = 1 + 2x + 3x^2 + \ldots + (r+1)x^r$

$\therefore$ Coefficient of x^r in $(1-x)^{-2}$ is $(r+1)$.

290 **(b)**

$$\frac{1}{(6-3x)^{1/3}} = (6-3x)^{-1/3} = 6^{-1/3}\left[1 - \frac{x}{2}\right]^{-1/3}$$

$$= 6^{-1/3}\left[1 + \left(-\frac{1}{3}\right)\left(-\frac{x}{2}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2.1}\left(-\frac{x}{2}\right)^2 + \ldots\right]$$

$$= 6^{-1/3}\left[1 + \frac{x}{6} + \frac{2x^2}{6^2} + \ldots\right]$$

291 **(a)**

The coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^{14}$ is ${}^{14}C_r$

It is given that

${}^{14}C_{r-1}, {}^{14}C_r, {}^{14}C_{r+1}$ are in A.P.

$\Rightarrow 2\,{}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1}$

$\Rightarrow 2 = \frac{{}^{14}C_{r-1}}{{}^{14}C_r} + \frac{{}^{14}C_{r+1}}{{}^{14}C_r}$

$\Rightarrow 2 - \frac{r}{15-r} + \frac{14-r}{r+1}$

$\Rightarrow 2(15-r)(r+1) = r^2 + r + 210 - 29\,r + r^2$

$\Rightarrow 4\,r^2 - 56\,r + 180 = 0$

$\Rightarrow r^2 - 14r + 45 = 0 \Rightarrow r = 5, 9$

292 **(b)**

If n id odd, then numerically the greatest coefficient in the expansion of $(1-x)^n$ is ${}^nC_{\frac{n-1}{2}}$ or, ${}^nC_{\frac{n+1}{2}}$

Therefore, in case of $(1-x)^{21}$ the numerically greatest coefficient is ${}^{21}C_{10}$ or, ${}^{21}C_{11}$

Numerically greatest term

$= {}^{21}C_{11}\,x^{11}$ or, ${}^{21}C_{10}x^{10}$

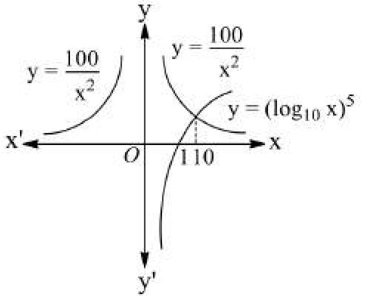
$\therefore\ {}^{21}C_{11}x^{11} > {}^{21}C_{12}\,x^{12}$ and ${}^{21}C_{10}\,x^{10} > {}^{21}C_9\,x^9$

$\Rightarrow \frac{21!}{10!\,11!} > \frac{21!}{9!\,12!}\,x$ and $\frac{21!}{11!\,10!}x > \frac{21!}{9!\,12!}$

$\Rightarrow \frac{6}{5} > x$ and $x < \frac{5}{6} \Rightarrow x \in (5/6, 6/5)$

293 **(d)**

Note that for $\log_{10} x$ to be defined, $x > 0$



We have, $T_6 = T_{5+1} = {}^8C_5\left(\frac{1}{x^{8/3}}\right)^{8-5}(x^2\log_{10}x)^5$

$\Rightarrow 5600 = \frac{8!}{5!\,3!}\left(\frac{1}{x^8}\right)x^{10}(\log_{10}x)^5$

$\Rightarrow 5600 = 56x^2(\log_{10}x)^5$

$\Rightarrow 100 = x^2(\log_{10}x)^5$

$\Rightarrow \frac{100}{x^2} = (\log_{10}x)^5$

Let $y = \frac{100}{x^2}$

$\therefore\ y = (\log_{10}x)^5$

From the figure it is clear that curves intersect in just one point.

This point is $(10, 1)$

Therefore, $x = 10$

294 **(b)**

We have,

$(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$

$= (1+x^2)^{40}(x^2+1)^{-10}x^{10}$

$\Rightarrow (1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5} = (1+x^2)^{30}x^{10}$

$\therefore$ Coeff of x^{20} in the expansion of $(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$

$=$ Coefficient of x^{20} in $(1+x^2)^{30}\cdot x^{10}$

$=$ Coefficient of x^{10} in $(1+x^2)^{30} = {}^{30}C_5 = {}^{30}C_{25}$

295 **(d)**

Let $P(n) = n^3 + 2n$

$\Longrightarrow P(1) = 1 + 2 = 3$

$\Longrightarrow P(2) = 8 + 4 = 12$

$\Longrightarrow P(3) = 27 + 6 = 33$

Here, we see that all these number are divisible by 3

296 **(c)**

$(1+x)^m(1-x)^n$

$= \left(1 + mx + \frac{m(m-1)x^2}{2!} + \dots\right)\left(1 - nx + \frac{n(n-1)}{2!}x^2 - \dots\right)$

$= 1 + (m-n)x + \left[\frac{n^2-n}{2} - mn + \frac{(m^2-m)}{2}\right]x^2 \dots$

Given, $m - n = 3 \Longrightarrow n = m - 3$

and $\frac{n^2-n}{2} - mn + \frac{m^2-m}{2} = -6$

$\Longrightarrow \frac{(m-3)(m-4)}{2} - m(m-3) + \frac{m^2-m}{2} = -6$

$\Longrightarrow m^2 - 7m + 12 - 2m^2 + 6m + m^2 - m + 12 = 0$

$\Longrightarrow -2m + 24 = 0 \Longrightarrow m = 12$

297 **(d)**

We have,

$(1 + x + x^2 + x^3)^n$

$= (1+x)^n(1+x^2)^n$

$= (C_0 + C_1\,x + C_2\,x^2 + \dots + C_n\,x^n)(C_0 + C_1x^2 + \dots + C_n\,x^{2n})$

$\therefore$ Coefficient of $x^4 = C_0C_2 + C_2C_1 + C_4C_0$

$= {}^nC_2 + {}^nC_2\cdot{}^nC_1 + {}^nC_4$

298 **(b)**

Let, $S = 1 + \frac{1}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2} + \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{8}{9}\cdot\frac{1}{2^3} + \dots\infty$

and we know that

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{(n(n-1)(n-2)}{3!}x^2 + \dots\infty$

On comparing these two, we get

$nx = \frac{2}{3}\cdot\frac{1}{2}$ ... (i)

and $\frac{n(n-1)}{2\cdot 1}x^2 = \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2}$ ... (ii)

from Eqs. (i) and (ii),

$\Rightarrow \frac{\frac{n(n-1)}{2\cdot 1}}{n^2} = \frac{\frac{2}{3}\times\frac{5}{6}\times\frac{1}{4}}{\frac{2}{3}\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}}$

$$\Rightarrow \quad \frac{n-1}{2n} = \frac{5}{4}$$

$$\Rightarrow \quad 5n = 2n - 2$$

$$\Rightarrow \quad n = -\frac{2}{3}$$

On putting value of n in Eq. (i), we get

$$x = -\frac{1}{2}$$

$$\therefore \text{Sum of series} = \left(1 - \frac{1}{2}\right)^{-\frac{2}{3}} = (4)^{1/3}$$

299 **(b)**

Here, $T_5 + T_6 = 0$

$$\Rightarrow {}^nC_4 a^{n-4}(-2b)^4 + {}^nC_5 a^{n-5}(-2b)^5 = 0$$

$$\Rightarrow 16 \cdot {}^nC_4 a^{n-4} b^4 = 32\, {}^nC_5 a^{n-5} b^5$$

$$\Rightarrow \frac{{}^nC_5}{{}^nC_4} \cdot \frac{a^{n-5}b^5}{a^{n-4}b^4} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{b}{a} = \frac{1}{2} \cdot \frac{{}^nC_4}{{}^nC_5}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{2 \cdot \frac{n!}{5!(n-5)!}}{\frac{n!}{4!(n-4)!}}$$

$$= \frac{4!\,(n-4)!}{5!\,(n-5)!} \times 2 = \frac{2(n-4)}{5}$$

300 **(c)**

Given, $(1+x)^m(1-x)^n$

$$= \left(1 + mx + m\frac{(m-1)}{2!}x^2 + \cdots\right)$$

$$\left(1 - nx + \frac{n(n-1)}{2!}x^2 - \cdots\right)$$

$$= 1 + (m-n)x + \left[\frac{n^2-n}{2} - mn + \frac{m^2-m}{2}\right]x^2 + \cdots$$

Also, given $m - n = 3 \Rightarrow n = m - 3$

and $\quad \frac{n^2-n}{2} - mn + \frac{m^2-m}{2} = -6$

$$\Rightarrow \quad \frac{(m-3)(m-4)}{2} - m(m-3) + \frac{m^2-m}{2} = -6$$

$$\Rightarrow m^2 - 7m + 12 - 2m^2 + 6m + m^2 - m + 12 = 0$$

$$\Rightarrow \quad -2m + 24 = 0 \Rightarrow m = 12$$

301 **(b)**

Given sum of the coefficient = 1024

ie, $\quad 2^n = 1024 = 2^{10}$

$\Rightarrow n = 10$

Since, n is even, so greatest coefficient

$= {}^nC_{n/2} = {}^{10}C_5 = 252$

302 **(b)**

We have,

$5^{99} = 5^3 \times 5^{96}$

$$= (13 \times 9 + 8)\{1 + {}^{24}C_1(13 \times 48) + \cdots + {}^{24}C_{24}(13 \times 48)^{24}\}$$

$$= (13 \times 9 + 8) + (13 \times 9 + 8)\{{}^{24}C_1(13 \times 48) + \cdots + {}^{24}C_{24}(13 \times 48)^{24}\}$$

$= 8 + 13 \times$ An integer

Hence, remainder = 8

303 **(c)**

General term of $(3+2x)^{74}$ is

$$T_{r+1} = {}^{74}C_r (3)^{74-r} 2^r x^r$$

Let two consecutive terms are T_{r+1}th and T_{r+2}th terms

According to the given condition,

Coefficient of T_{r+1} = Coefficient of T_{r+2}

$$\Rightarrow {}^{74}C_r 3^{74-r} 2^r = {}^{74}C_{r+1} 3^{74-(r+1)} 2^{r+1}$$

$$\Rightarrow \frac{{}^{74}C_{r+1}}{{}^{74}C_r} = \frac{3}{2} \Rightarrow \frac{74-r}{r+1} = \frac{3}{2}$$

$$\Rightarrow 148 - 2r = 3r + 3 \Rightarrow r = 29$$

Hence, two consecutive terms are 30 and 31.

304 **(b)**

Given expansion is $\left(\frac{2}{3}x - \frac{3}{2x}\right)^n$

$$\therefore T_4 = {}^nC_3 \left(\frac{2}{3}x\right)^{n-3} \left(-\frac{3}{2x}\right)^3$$

$$= {}^nC_3 \left(\frac{2}{3}\right)^{n-6} x^{n-6}(-1)^3$$

Since, it is independent of x

$\therefore n - 6 = 0 \Rightarrow n = 6$

305 **(b)**

General term, $T_{r+1} = {}^{15}C_r (x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r$

$= {}^{15}C_r x^{60-7r}(-1)^r$

For the coefficient of x^{3r} put

$$60 - 7r = 32 \Rightarrow r = 4$$

Now, coefficient of x^{32} in $\left(x^4 - \frac{1}{x^3}\right)^{15} =$

${}^{15}C_4(-1)^4 = {}^{15}C_4$

306 **(d)**

The number of terms in $(a+b+c)^{12}$

$= {}^{12+2}C_2 = {}^{14}C_2 = 91$

307 **(a)**

Coefficient of x^7 in $(1 + 3x - 2x^3)^{10}$

$$= \sum \frac{10!}{n_1!\,n_2!\,n_3!}(1)^{n_1}(3)^{n_2}(-2)^{n_3}$$

Where, $n_1 + n_2 + n_3 = 10, \;\; n_2 + 3n_3 = 7$

Different possibilities are as follows

n_1	n_2	n_3
3	7	0
5	4	1
7	1	2

$\therefore$ Coefficient of $x^7 = \frac{10!}{3!7!}(1)^3(3)^7(-2)^0$

$+\frac{10!}{5!\ 4!\ 1!}(1)^5(3)^4(-2)^1$

$+\frac{10!}{7!\ 1!\ 2!}(1)^7(3)^1(-2)^2$

$= 62640$

308 **(b)**

The general term in the expansion of $\left(x-\frac{1}{x}\right)^{18}$ is

$$T_{r+1} = {}^{18}C_r(x)^{18-r}\left(-\frac{1}{x}\right)^r$$

Here, $n = 18$

$\therefore$ the middle term is T_{9+1}, where $r = 9$

$\therefore \quad T_{9+1} = {}^{18}C_9(-1)^9x^{18-2r}$

$= -{}^{18}C_9x^{18-18} = -{}^{18}C_9$

309 **(d)**

General term

$T_{r+1} = (-1)^r\,{}^{11}C_r\left(\frac{2\sqrt{x}}{5}\right)^{11-r}\left(\frac{1}{2x^{3/2}}\right)^r$

$= \frac{2^{11-2r}}{5^{11-r}}(-1)^r\,{}^{11}C_r x^{\frac{11-r}{2}-\frac{3r}{2}}$

For term independent of x, put $\frac{11-r}{2} - \frac{3r}{2} = 0$

$\Rightarrow \frac{11-4r}{2} = 0 \Rightarrow r = \frac{11}{4} \notin N$

$\therefore$ There is no term which is independent of x.

310 **(b)**

$a[C_0 - C_1 + C_2 - C_3 + \cdots (-1)^n \cdot C_n]$

$+[C_1 - 2C_2 + 3C_3 - \cdots + (-1)^{n-1}nC_n]$

$= a0 + 0 = 0$

312 **(a)**

Let $(6\sqrt{6}+14)^{2n+1} = I + F$, where $I \in N$ and $0 < F < 1$

Also, let $G = (6\sqrt{6}-14)^{2n+1}$. Then, $0 < G < 1$

Clearly,

$I + F - G$ is an even integer

$\Rightarrow F = G$

$\Rightarrow I$ is an even integer

313 **(a)**

The sum of the coefficients is obtained by putting $x = y = 1$ in $(5x-4y)^n$. So, required sum $= 1$

314 **(a)**

Now, $\left[\frac{(x+1)}{x^{2/3}-x^{1/3}+1} - \frac{(x-1)}{x-x^{1/2}}\right]^{10}$

$= \left[\frac{(x^{1/3})^3+1^3}{x^{2/3}-x^{1/3}+1} - \frac{((\sqrt{x})^2-1)}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$

$= [x^{1/3} + 1 - (x^{-1/2} + 1]^{10}$

$= [x^{1/3} + x^{-1/2}]^{10}$

$\therefore T_{r+1} = {}^{10}C_r(x)^{\frac{10-r}{3}}(-x^{-1/2})^r$

$= {}^{10}C_r(-1)^r x^{\frac{20-5r}{6}}$

For the term independent of x

Put $\frac{20-5r}{6} = 0$

$\Rightarrow r = 4$

$\therefore$ Required coefficient $= {}^{10}C_4 = 210$

315 **(c)**

Putting the values of C_0, C_2, C_4...., we get

$1 + \frac{n(n-1)}{3\cdot 2!} + \frac{n(n-1)(n-2)(n-3)}{5\cdot 4!} + \cdots$

$= \frac{1}{n+1}\left[(n+1) + \frac{(n+1)n(n-1)}{3!} + \frac{(n+1)n(n-1)(n-2)(n-3)}{5!} + \cdots\right]$

Put $n+1 = N$

$= \frac{1}{N}\left[N + \frac{N(N-1)(N-2)}{3!}\right] + \frac{N(N-1)(N-2)(N-3)(N-4)}{5!} + \cdots$

$= \frac{1}{N}[{}^NC_1 + {}^NC_3 + {}^NC_5 + \cdots]$

$= \frac{1}{N}[2^{N-1}] = \frac{2^n}{n+1}\ [\because N = n+1]$

316 **(b)**

We have,

$\frac{(1+x)^n}{1-x} = (1+x)^n(1-x)^{-1}$

$\Rightarrow \frac{(1+x)^n}{1-x} = ({}^nC_0\,x^n + {}^nC_1\,x^{n-1} + \cdots + {}^nC_{n-1}x + {}^nC_nx^0) \times (1 + x + x^2 + x^3 + \cdots + x^n + \cdots)$

$\therefore$ Coefficient of x^n in $\frac{(1+x)^n}{1-x}$

$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$

317 **(d)**

Given that, $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$

On putting $x = 1$ and $x = -1$ and adding the results, we get

$64 = 2(1 + a_2 + a_4 + \ldots + a_{12})$

$\therefore\ a_2 + a_4 + a_6 + \ldots + a_{12} = 31$

318 **(b)**

$\because\ (1+x)^n = \sum_{r=0}^{n}{}^nC_rx^r = \sum_{r=0}^{n}a_rx^r$ (given)

$\therefore\ a_r = {}^nC_r$

Also, $b_r = 1 + \frac{a_r}{a_{r-1}} = 1 + \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{{}^{n+1}C_r}{{}^nC_{r-1}}$

$b_r = \left(\frac{n+1}{r}\right)$

$\because \prod_{r=1}^{n} b_r = \prod_{r=1}^{n}\left(\frac{n+1}{r}\right) = \frac{(n+1)^n}{n!}$

$= \frac{(101)^{100}}{100!}$ (given)

$\therefore n = 100$

319 **(c)**

Coefficient of $x^2y^3z^4 = \frac{9!}{2!\,3!\,4!}a^2b^3c^4$

$= 1260a^2b^3c^4$

320 **(c)**

$T_{r+1} = {}^9C_r(x^2)^{9-r}\left(-\frac{1}{x}\right)^r$

$= {}^9C_r x^{18-2r-r}(-1)^r$

For term independent of x, put $18 - 2r - r = 0 \Rightarrow r = 0$

$\therefore$ Constant term, $T_7 = {}^9C_6(-1)^6 = 84$

321 **(c)**

We have,

$\sum_{k=1}^{\infty} k\left(1+\frac{1}{n}\right)^{k-1}$

$= \sum_{k=1}^{\infty} k\,x^{k-1}$, where $x = 1 + \frac{1}{n}$

$= 1 + 2x + 3x^2 + 4x^3 + \cdots$ to ∞

$= (1-x)^{-2} = \left(-\frac{1}{n}\right)^{-2} = n^2$

322 **(c)**

Let $(r+1)$th, $(r+2)$th and $(r+3)$th be three consecutive terms.

Then, ${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 1:7:42$

Now, $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{1}{7} \Rightarrow \frac{r+1}{n-r} = \frac{1}{7} \Rightarrow n - 8r = 7$

and $\frac{{}^nC_{r+1}}{{}^nC_{r+2}} = \frac{7}{42}$

$\Rightarrow \frac{r+2}{n-r-1} = \frac{1}{6}$

$\Rightarrow n - 7r = 13$ ...(ii)

On solving Eqs. (i) and (ii), we get $n = 55$

323 **(c)**

We have,

$(1+3\sqrt{2}\,x)^9 + (1-3\sqrt{2}\,x)^9$

$= 2\left\{{}^9C_0 + {}^9C_2(3\sqrt{2}\,x)^2 + \cdots + {}^9C_8(3\sqrt{2}\,x)^8\right\}$

Clearly, there are 5 terms in the above expansion

324 **(c)**

Given that, ${}^nC_6 = {}^nC_{12} \Rightarrow {}^nC_{n-6} = {}^nC_{12}$

$\Rightarrow n - 6 = 12 \Rightarrow n = 18$

325 **(c)**

The general term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is

$T_{r+1} = (-1)^r\,{}^{12}C_r \cdot 2^{12-r} \cdot x^{24-3r}$

The term independent of x, put $24 - 3r = 0$

$\Rightarrow r = 8$

$\therefore$ In the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$, the term independent of x is 9th term.

327 **(b)**

$\because 4^n = (1+3)^n$

$= 1 + 3n + \frac{n(n-1)}{2!}3^2 + \cdots$

$\Rightarrow 4^n - 3n - 1 = 3^2\left[\frac{n(n-1)}{2!} + ..\right]$

It is clear from above that $4^n - 3n - 1$is divisible by 9.

328 **(b)**

On putting $x = -1$in

$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + \ldots + {}^{20}C_{10}x^{10} + \cdots + {}^{20}C_{10}x^{10}$,

we get

$0 = {}^{20}C_0 - {}^{20}C_1 + \cdots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \cdots + {}^{20}C_{20}$

$\Rightarrow 0 = {}^{20}C_0 - {}^{20}C_1 + \cdots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_9 + \cdots + {}^{20}C_0$

$\Rightarrow 0 = 2({}^{20}C_0 - {}^{20}C_1 + \cdots - {}^{20}C_9) + {}^{20}C_{10}$

$\Rightarrow {}^{20}C_{10} = 2({}^{20}C_0 - {}^{20}C_1 + \cdots + {}^{20}C_{10})$

$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + \ldots + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$

329 **(d)**

Now, $(\sqrt{3}+1)^5 = (\sqrt{3})^5 + {}^5C_1(\sqrt{3})^4 + {}^5C_2(\sqrt{3})^3 + {}^5C_3(\sqrt{3})^2 + {}^5C_4(\sqrt{3}) + {}^5C_5$

$= 9\sqrt{3} + 45 + 30\sqrt{3} + 30 + 5\sqrt{3} + 1$

$= 76 + 44\sqrt{3}$

$\therefore \left[(\sqrt{3}+1)^5\right] = [76 + 44\sqrt{3}]$

$= [76] + [44 \times 1.732]$

$= 76 + 76 = 152$

330 **(c)**

The given sigma expansion $\sum_{m=0}^{100} {}^{100}C_m\,(x-3)^{100-m} \cdot 2^m$ can be rewritten as

$[(x-3)+2]^{100} = (x-1)^{100} = (1-x)^{100}$

$\therefore x^{53}$ will occur in T_{54}

$\Rightarrow T_{54} = {}^{100}C_{53}(-x)^{53}$

$\therefore$ Required coefficient is$-{}^{100}C_{53}$.

331 **(a)**

$T_4 = {}^nC_3(ax)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$ [given]

$\Rightarrow {}^nC_3a^{n-3}x^{n-6} = \frac{5}{2}$

$\Rightarrow n-6=0 \quad$ [$\because$ RHS is independent of x]

$\Rightarrow n=6$

On putting $n=6$ in Eq. (i), we get

${}^6C_3 a^3=\frac{5}{2} \Rightarrow a^3=\frac{1}{8} \Rightarrow a=\frac{1}{2}$

332 **(a)**

We have,

$$\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)C_rC_s=\sum_{r=0}^{n}2rC_r^2 + 2\sum_{0\le r<s\le n}\sum (r+s)\,C_rC_s$$

$$\Rightarrow \sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)C_rC_s=2\sum_{r=0}^{n}r\cdot C_r^2 + 2\sum_{0\le r<s\le n}\sum (r+s)\,C_rC_s$$

$$\Rightarrow n\cdot 2^{2n}=2\cdot\left(\frac{n}{2}\,{}^{2n}C_n\right) + 2\sum_{0\le r<s\le n}\sum (r+s)\,C_rC_s$$

$$\Rightarrow \sum_{0\le r<s\le n}\sum (r+s)\,C_rC_s=\frac{1}{2}[n\cdot 2^{2n}-n\cdot {}^{2n}C_n]$$

$$=\frac{n}{2}\left[2^{2n}-\frac{2n}{n}\,{}^{2n-1}C_{n-1}\right]$$

$$=n[2^{2n-1}-{}^{2n-1}C_{n-1}]$$

333 **(a)**

Suppose x^7 occurs in $(r+1)^{th}$ term in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$

We have,

$$T_{r+1}={}^{11}C_r(a\,x^2)^{11-r}\left(\frac{1}{b\,x}\right)^r$$

$$={}^{11}C_r\,a^{11-r}\,b^{-r}x^{22-3r}$$

This will contain x^7, if

$22-3r=7 \Rightarrow r=5$

$\therefore$ Coefficient of x^7 in $(ax^2+b^{-1}x^{-1})^{11}$

$={}^{11}C_5a^6b^{-5}$

Let x^{-7} occur in $(s+1)^{th}$ term of the expansion of $\left(ax-\frac{1}{b\,x^2}\right)^{11}$

We have,

$$T_{s+1}={}^{11}C_s\,(a\,x)^{11-s}\left(-\frac{1}{b\,x^2}\right)^s$$

$\Rightarrow T_{s+1}={}^{11}C_s\,a^{11-s}b^{-s}(-1)^s\,x^{11-3s}$

This is contain x^{-7}, if

$11-3s=-7 \Rightarrow s=6$

$\therefore$ Coefficient of x^{-7} in $(ax-b^{-1}x^{-2})^{11}$

$={}^{11}C_6\,a^5b^{-6}$

It is given that

${}^{11}C_6\,a^5b^{-6}={}^{11}C_5a^6b^{-5} \Rightarrow ab=1$

334 **(a)**

We have,

$$\frac{1}{(1-x)(3-x)}=\frac{1}{2}\left(\frac{1}{1-x}-\frac{1}{3-x}\right)$$

$$\Rightarrow \frac{1}{(1-x)(3-x)}=\frac{1}{2}\{(1-x)^{-1}-(3-x)^{-1}\}$$

$$\Rightarrow \frac{1}{(1-x)(3-x)}=\frac{1}{2}\left\{(1-x)^{-1}-\frac{1}{3}\left(1-\frac{x}{3}\right)^{-1}\right\}$$

$\therefore$ Coefficient $x^n=\frac{1}{2}\left\{1-\frac{1}{3}\cdot\frac{1}{3^n}\right\}=\frac{1}{2}\frac{(3^{n+1}-1)}{3^{n+1}}$

335 **(a)**

For greatest term in $(x+a)^a$is

$$\frac{n-r+1}{r}\left|\frac{a}{x}\right|\ge 1$$

$$\Rightarrow \quad \frac{54-r+1}{r}\left|\frac{3x}{1}\right|\ge 1$$

$\Rightarrow 55-r\ge r \Rightarrow r=27 \quad \left[\because x=\frac{1}{3}\right]$

$\therefore$ Greatest term in the expansion of $(1+3x)^{54}$ is T_{28}.

336 **(b)**

We have,

$$T_4={}^5C_3\left(\frac{1}{x}\right)^{5-3}(x\tan x^3)=10x\tan^3 x$$

and $T_2={}^5C_2\left(\frac{1}{x}\right)^{5-1}(x\tan x)=\frac{5\tan x}{x^3}$

Given $\frac{T_4}{T_2}=\frac{2}{27}\pi^4 \Rightarrow 2x^4\tan^2 x=\frac{2}{27}\pi^4$

$\Rightarrow x^2\tan x=\pm\frac{1}{3\sqrt{3}}\pi^2$

It is possible (from among the answers) when $x=\pm\frac{\pi}{3}$

337 **(d)**

We have,

$32^{32}=(2^5)^{32}=2^{160}=(3-1)^{160}$

$\Rightarrow 32^{32}={}^{160}C_0\,3^{160}-{}^{160}C_1\cdot 3^{159}+\cdots - {}^{160}C_{159}\cdot 3+{}^{160}C_{160}3^0$

$\Rightarrow 32^{32}=({}^{160}C_0\cdot 3^{160}-{}^{160}C_1\cdot 3^{159}+\cdots - {}^{160}C_{159}\cdot 3)+1$

$\Rightarrow 32^{32}=3m+1$, where $m\in N$

$\therefore 32^{(32)^{(32)}}=(32)^{3m+1}=(2^5)^{3m+1}=2^{15m+5}$

$=2^{3(5m+1)}\cdot 2^2$

$\Rightarrow 32^{(32)^{(32)}}=(2^3)^{5m+1}\cdot 2^2=(7+1)^{5m+1}\times 4$

$\Rightarrow 32^{(32)^{(32)}}=\{{}^{5m+1}C_0\,7^{5m+1}+{}^{5m+1}C_1\,7^{5m}+\cdots + {}^{5m+1}C_{5m}\,7^1+{}^{5m+1}C_{5m+1}\cdot 7^0\}\times 4$

$\Rightarrow 32^{(32)^{(32)}}=(7n+1)\times 4$,

where $n={}^{5m+1}C_0 7^{5m+1}+\cdots+{}^{5m+1}C_{5m}7$

$\Rightarrow 32^{(32)^{(32)}}=28n+4$

Thus, when $32^{(32)^{(32)}}$ is divided by 7, the remainder is 4

338 **(a)**

Since, $\left(1+\frac{1}{n}\right)^n < 3$ for $\forall n \in N$

Now, $\frac{(1001)^{999}}{(1000)^{1000}} = \frac{1}{1001} \cdot \left(\frac{1001}{1000}\right)^{1000}$

$= \frac{1}{1001}\left(1+\frac{1}{1000}\right)^{1000} < \frac{1}{1001} \cdot 3 < 1$

$(1001)^{999} < (1000)^{1000}$

$\therefore B < A$

339 **(a)**

$\therefore T_r = 10_{C_{r-1}} \left(\frac{x}{3}\right)^{11-r} \left(\frac{-2}{x^2}\right)^{r-1}$

$= 10_{C_{r-1}} (x)^{13-3r} (3)^{-11+r} (-1)^{r-1} (2)^{r-1}$

For x^4, we put $13 - 3r = 4 \Rightarrow r = 3$

340 **(a)**

Given, $\left[\sqrt{\frac{x}{3}} + \frac{\sqrt{3}}{x^2}\right]^{10}$

General term, $T_{r+1} = {}^{10}C_r \left(\frac{x}{3}\right)^{\frac{1}{2}(10-r)} \left(\frac{\sqrt{3}}{x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^{10}C_r \left(\frac{1}{3}\right)^{\frac{10-r}{2}} (\sqrt{3})^r x^{\frac{1}{2}(10-r)-2r}$

For term independent of x put

$\frac{1}{2}(10-r) - 2r = 0$

$\Rightarrow \quad r = 2$

$\therefore T_{2+1} = T_3 = {}^{10}C_2 \left(\frac{1}{3}\right)^{\frac{8}{2}} (\sqrt{3})^2$

$\Rightarrow = 45 \times \frac{1 \times 3}{81} = \frac{5}{3}$

341 **(c)**

We have,

$T_{r+1} = {}^{15}C_r (x^2)^{15-r} \left(\frac{2}{x}\right)^r = {}^{15}C_r\, x^{30-3r} \cdot 2^r$

If T_{r+1} contains x^{15}, then

$30 - 3r = 15 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^{15} = {}^{15}C_5(2^5)$

If T_{r+1} does not contain x, then

$30 - 3r = 0 \Rightarrow r = 10$

$\therefore$ Coefficient of $x^0 = {}^{15}C_{10}(2^{10})$

Hence, required ratio $= \frac{{}^{15}C_5(2^5)}{{}^{15}C_{10}(2^{10})} = \frac{1}{32}$

342 **(d)**

We have,

${}^{40}C_0 + {}^{40}C_1 + {}^{40}C_2 + \cdots + {}^{40}C_{20}$

$= \frac{1}{2}[2 \cdot {}^{40}C_0 + 2 \cdot {}^{40}C_1 + 2 \cdot {}^{40}C_2 + \cdots + 2 \cdot {}^{40}C_{20}]$

$= \frac{1}{2}[({}^{40}C_0 + {}^{40}C_{40}) + ({}^{40}C_1 + {}^{40}C_{0=39}) + \cdots + ({}^{40}C_{19} + {}^{40}C_{21}) + 2\,{}^{40}C_{20}]$

$= \frac{1}{2}[\{{}^{40}C_0 + {}^{40}C_1 + {}^{40}C_2 + \cdots + {}^{40}C_{19} + {}^{40}C_{20} + {}^{40}C_{21}\} + {}^{40}C_{20}]$

$= \frac{1}{2}\left[2^{40} + \frac{40!}{(20\,!)^2}\right] = 2^{39} + \frac{1}{2}\frac{40!}{(20!)^2}$

343 **(d)**

We have,

$\frac{1+x^2}{1+x} = (1+x^2)(1+x)^{-1}$

$= (1+x^2)(1 - x + x^2 - x^3 + x^4 - x^5 + \cdots)$

$\therefore$ Coefficient of x^5 in $\left(\frac{1+x^2}{1+x}\right) = -1 - 1 = -2$

344 **(a)**

Genaral term $T_{r+1} = {}^{10}C_r \left(\frac{1}{3}x^{1/2}\right)^{10-r} (x^{-1/4})^r$

$= {}^{10}C_r \frac{1}{3^{10-r}} x^{5-3r/4}$

For the coefficient of x^2 put

$5 - \frac{3r}{4} = 2$

$\Rightarrow \quad r = 4$

$\therefore$ Coefficient of $x^2 = {}^{10}C_4 \frac{1}{3^{10-4}} = \frac{70}{243}$

345 **(d)**

The 14th term in the expansion of $\left(\frac{3\sqrt{x}}{7} - \frac{5}{2x\sqrt{x}}\right)^{13n}$ is

$T_{14} = {}^{13n}C_{13} \left(\frac{3}{7}x^{1/2}\right)^{13n-13} (-1)^{13} \left(\frac{5}{2}x^{-3/2}\right)^{13}$

$= {}^{13n}C_{13} \left(\frac{3}{7}\right)^{13n-13} (-1)^{13} \left(\frac{5}{2}\right)^{13} x^{\frac{13n-13}{2} - \frac{39}{2}}$

For this term to be independent of x, we put

$13n - 52 = 0$

$\Rightarrow \quad n = 4$

346 **(b)**

Here, the greatest coefficient is ${}^{2n}C_n$

$\therefore\ {}^{2n}C_n x^n > {}^{2n}C_{n+1} x^{n-1} \Rightarrow x > \frac{n}{n+1}$

and ${}^{2n}C_n x^n > {}^{2n}C_{n-1} x^{n-1} \Rightarrow x < \frac{n+1}{n}$

$\therefore x$ must lie in the interval $\left(\frac{n}{n+1}, \frac{n+1}{n}\right)$

347 **(d)**

$(1 + a - b + c)^9$

$= \sum \frac{9!}{x_1!\, x_2!\, x_3!\, x_4!} \cdot (1)^{x_1} (a)^{x_2} (-b)^{x_3} (c)^{x_4}$

$\Rightarrow$ Coefficient of $a^3b^4c = \frac{9!}{1!\,3!\,4!\,1!} = \frac{9!}{3!\,4!}$

348 **(b)**

We have, $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1+x+x^2)^{n-1}(1+2x)$

$= a_1 + 2a_2x + 3a_3x^2 + \ldots + 2na_{2n}x^{2n-1}$

On putting $x = -1$, we get

$n(1-1+1)^{n-1}(1-2)$

$= a_1 - 2a_2 + 3a_3 - \ldots - 2na_{2n}$

$\Rightarrow a_1 - 2a_2 + 3a_3 - \ldots - 2na_{2n} = -n$

350 **(b)**

We have,

$T_4 = 200$

$\Rightarrow {}^6C_3\left\{\sqrt{x^{\frac{1}{\log x+1}}}\right\}^3\left(x^{\frac{1}{12}}\right)^3 = 200$

$\Rightarrow 20\, x^{\frac{3}{2(\log x+1)}+\frac{1}{4}} = 200$

$\Rightarrow x^{\frac{3}{2(\log x+1)}+\frac{1}{4}} = 10$

$\Rightarrow \frac{3}{2(\log x+1)} + \frac{1}{4} = \log_x 10$

$\Rightarrow \frac{3}{2(y+1)} + \frac{1}{4} = \frac{1}{y}$, where $y = \log_{10} x$

$\Rightarrow \frac{6+y+1}{4(y+1)} = \frac{1}{y}$

$\Rightarrow 7y + y^2 = 4y + 4$

$\Rightarrow y^2 + 3y - 4 = 0$

$\Rightarrow (y+4)(y-1) = 0$

$\Rightarrow y = -4, y = 1$

$\Rightarrow \log_{10} x = -4$ or, $\log_{10} x = 1$

$\Rightarrow x = 10^{-4}$ or, $x = 10^1 \Rightarrow x = 10$ $[\because x > 1]$

351 **(a)**

We have,

$\{(1+x)^6 + (1+x)^7 + \cdots + (1+x)^{15}\}$

$= (1+x)^6\left\{\frac{1-(1+x)^{10}}{1-(1+x)}\right\}$

$= (1+x)^6\left\{\frac{1-(1+x)^{10}}{-x}\right\}$

$= \frac{1}{x}\{(1+x)^{16} - (1+x)^6\}$

$\therefore$ Coefficient of x^6 in $\{(1+x)^6 + (1+x)^7 + \cdots + (1+x)^{15}\}$

$=$ Coeff. of x^7 in $\{(1+x)^{16} - (1+x)^6\} = {}^{16}C_7 = {}^{16}C_9$

352 **(d)**

Let $S = 1 + \frac{1}{5} + \frac{1\cdot 3}{5\cdot 10} + \frac{1\cdot 3\cdot 5}{5\cdot 10\cdot 15} + \ldots$

On comparing with

$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$, we get

$\Rightarrow nx = \frac{1}{5}$ and $\frac{n(n-1)x^2}{2!} = \frac{1\cdot 3}{5\cdot 10}$

$\Rightarrow \quad n = -\frac{1}{2}$

and $\quad x = -\frac{2}{5}$

$\therefore$ sum $= \left(1-\frac{2}{5}\right)^{-1/2} = \left(\frac{3}{5}\right)^{-1/2} = \sqrt{\frac{5}{3}}$

353 **(d)**

$A^2 = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$

$A^3 = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}$

… … … … … …

… … … … … …

$A^n = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ Can be verified by induction. Now, taking option

(b) $\begin{bmatrix}1 & 0\\n & 1\end{bmatrix} = \begin{bmatrix}n & 0\\n & n\end{bmatrix} + \begin{bmatrix}n-1 & 0\\0 & n-1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0\\n & 1\end{bmatrix} \neq \begin{bmatrix}2n-1 & 0\\1 & 2n-1\end{bmatrix}$

(d) $nA - (n-1)I = \begin{bmatrix}n & 0\\n & n\end{bmatrix} - \begin{bmatrix}n-1 & 0\\1 & n-1\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\n & 1\end{bmatrix} = A^n$

354 **(c)**

$T_{r+1} = \sqrt{3}\cdot {}^{20}C_r\left(\frac{1}{\sqrt{3}}\right)^r$

and $T_r = \sqrt{3}\cdot {}^{20}C_{r-1}\left(\frac{1}{\sqrt{3}}\right)^{r-1}$

Now, $\frac{T_{r+1}}{T_r} = \frac{20-r+1}{r}\left(\frac{1}{\sqrt{3}}\right)$

Since, $T_{r+1} \geq T_r \Longrightarrow 20 - r + 1 \geq \sqrt{3}r$

$\Rightarrow r \leq \frac{21}{\sqrt{3}+1} = \frac{21}{2\cdot 73} \Longrightarrow r \leq 7.692$

$\Rightarrow r = 7$

$\therefore$ The greatest term is

$T_3 = \sqrt{3}\cdot {}^{20}C_7\left(\frac{1}{\sqrt{3}}\right)^7 = \frac{25840}{9}$

355 **(c)**

We have, $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$

$= \left(1+\frac{n}{1}\right)\left(1+\frac{\frac{n(n-1)}{2!}}{n}\right)\ldots\left(1+\frac{1}{n}\right)$

$= \frac{(1+n)}{1} \cdot \frac{(1+n)}{2} \cdot \frac{(1+n)}{3} \dots \frac{(1+n)}{n} = \frac{(n+1)^n}{n!}$

356 **(c)**

Given expression

$= 2[\,{}^5C_0 x^5 + {}^5C_2 x^3(x^3-1) + {}^5C_4 x(x^3-1)^2]$

$= 2[x^5 + 10\,x^3(x^3-1) + 5\,x(x^3-1)^2]$

$= 5\,x^7 + 10\,x^6 + x^5 - 10\,x^4 - 10x^3 + 5x,$

Which is a polynomial of degree 7

357 **(a)**

The sum of the coefficients is obtained by putting $x = 1$ in $(1 + x - 3\,x^2)^{2143}$

$\therefore$ required sum $= (1+1-3)^{2143} = -1$

358 **(d)**

We have, $C_0^2 - 2C_1^2 + 3C_2^2 - \ldots + (-1)^n(n+1)C_n^2$

$= [C_0^2 - C_1^2 + C_2^2 - \ldots + (-1)^n C_n^2]$

$- [C_1^2 - 2C_2^2 + 3C_3^2 - \ldots + (-1)^n n C_n^2]$

$= (-1)^{n/2} \frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!} \cdot -(-1)^{n/2-1} \cdot \frac{1}{2} n \frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}$

$= (-1)^{n/2} \cdot \frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!} \cdot \left(1 + \frac{n}{2}\right)$

Therefore, the value of the given expression is

$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!} \times (-1)^{n/2} \cdot \frac{(n)!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}\left(1 + \frac{n}{2}\right)$

$= (-1)^{n/2}(2+n)$

359 **(a)**

We have, $(1.002)^{12}$ or it can be rewritten as $(1+0.002)^{12}$

$\Rightarrow (1.002)^{12} = 1 + {}^{12}C_1(0.002) + {}^{12}C_2(0.002)^2 + {}^{12}C_3(0.002)^3 + \ldots$

We want the answer upto 4 decimal places and as such, we have left further expansion.

$\therefore (1.002)^{12} = 1 + 12(0.002) + \frac{12\cdot 11}{1\cdot 2}(0.002)^2 + \frac{12\cdot 11\cdot 10}{1\cdot 2\cdot 3}(0.002)^3 + \ldots$

$= 1 + 0.024 + 2.64 \times 10^{-4} + 1.76 \times 10^{-6} + \ldots$

$= 1.0242$

360 **(c)**

The general term in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is

$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$= {}^9C_r \left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$ ...(i)

Now, the coefficients of the terms x^0, x^{-1} and x^{-3} in $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is

For x^0, $18 - 3r = 0 \Rightarrow r = 6$

For x^{-1}, there exists no integer value of r

For x^{-3}, $18 - 3r = -3 \Rightarrow r = 7$

Now, the coefficient of the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$= 1.\ {}^9C_6(-1)^6\left(\frac{3}{2}\right)^{9-6}\left(\frac{1}{3}\right)^6 + 0$

$+ 2.\ {}^9C_7(-1)^7\left(\frac{3}{2}\right)^{9-7}\left(\frac{1}{3}\right)^7$

$= \frac{9.8.7}{1.2.3} \cdot \frac{3^3}{2^3} \cdot \frac{1}{3^6} + 2.\frac{9.8}{1.2}(-1)\frac{3^2}{2^2} \cdot \frac{1}{3^7}$

$= \frac{7}{18} - \frac{2}{27} = \frac{17}{54}$

361 **(c)**

We can write,

$aC_0 - (a+d)C_1 + (a+2d)C_2 - \ldots$ upto $(n+1)$ terms

$= a(C_0 - C_1 + C_2 - \ldots) + d(-C_1 + 2C_2 - 3C_3 + \ldots)$ ...(i)

We know,

$(1-x)^n = C_0 - C_1 x + C_2 x^2 - \ldots + (-1)^n C_n x^n$ ...(ii)

On differentiating Eq. (ii) w.r.t. x, we get

$-n(1-x)^{n-1} = -C_1 + 2C_2 x - \ldots (-1)^n C_n n x^{n-1}$ ...(iii)

On putting $x = 1$ in Eqs. (ii) and (iii), we get

$C_0 - C_1 + C_2 - \ldots + (-1)^n C_n = 0$ ...(iv)

and $-C_1 + 2C_2 - \ldots + (-1)^n nC_n = 0$ ...(v)

From Eq. (i),

$aC_0 - (a+d)C_1 + (a+2d)C_2 - \ldots$ upto $(n+1)$ terms

$= a \cdot 0 + d \cdot 0 = 0$ [from Eqs. (iv) and (v)]

362 **(c)**

Let $P(n) = 2^{3n} - 7n - 1$

$\therefore \quad P(1) = 0, P(2) = 49$

$P(1)$ and $P(2)$ are divisible by 49.

Let $P(k) \equiv 2^{3k} - 7k - 1 = 49I$

$\therefore P(k+1) \equiv 2^{3k+3} - 7k - 8$

$= 8(49I + 7k + 1) - 7k - 8$

$= 49(8I) + 49k = 49I_1$

Alternate

$P(n) = (1+7)^n - 7n - 1$

$= 1 + 7n + 7^2\frac{n(n-1)}{2!} + \cdots - 7n - 1$

$= 7^2\left(\frac{n(n-1)}{2!} + \cdots\right)$

363 **(d)**

Let $P(n) = 5^{2n+2} - 24n - 25$

For $\quad n = 1$

$P(1) = 5^4 - 24 - 25 = 576$

$P(2) = 5^6 - 24(2) - 25 = 15552$
$= 576 \times 27$
Here, we see that $P(n)$ is divisible by 576

364 **(c)**

Let $b = \sum_{r=0}^{n} \frac{r}{^nC_r} = \sum_{r=0}^{n} \frac{n-(n-r)}{^nC_r}$

$= n\sum_{r=0}^{n} \frac{1}{^nC_r} - \sum_{r=0}^{n} \frac{n-r}{^nC_r}$

$= na_n - \sum_{r=0}^{n} \frac{n-r}{^nC_{n-r}}$ $(\because {^nC_r} = {^nC_{n-1}})$

$= na_n - b$

$\Rightarrow 2b = na_n \Rightarrow b = \frac{n}{2}a_n$

365 **(d)**

We have,

$\frac{1}{\sqrt{4x+1}}\left\{\left(1+\frac{\sqrt{4x+1}}{2}\right)^2 - \left(1-\frac{\sqrt{4x+1}}{2}\right)^2\right\}$

$= \frac{1}{2^7\sqrt{4x+1}}\left[2\left\{ {^7C_1}\sqrt{4x+1} + {^7C_3}(\sqrt{4x+1})^3 + {^7C_5}(\sqrt{4x+1})^5 + {^7C_7}(\sqrt{4x+1})^7\right\}\right]$

$= \frac{1}{2^6}\{ {^7C_1} + {^7C_3}(4x+1) + {^7C_5}(4x+1)^2 + {^7C_7}(4x+1)^3\}$

Clearly, it is a polynomial of degree 3

366 **(a)**

In the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$,

$T_{r+1} = {^{11}C_r}(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$= {^{11}C_r}\frac{a^{11-r}}{b^r} \cdot x^{22-3r}$

For coefficient of x^7, put $22 - 3r = 7$

$\Longrightarrow r = 5$

$\therefore T_6 = {^{11}C_5}\frac{a^6}{b^5} \cdot x^7$

$\therefore$ Coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is

$11_{C_5}\frac{a^6}{b^5}$

Similarly, coefficient of x^{-7} in the expansion of $\left(ax + \frac{1}{bx}\right)^{11}$ is

${^{11}C_5}\frac{a^5}{b^6}$

Now, ${^{11}C_5}\frac{a^6}{b^5} = {^{11}C_6}\frac{a^5}{b^6}$

$\Longrightarrow ab = 1$

367 **(b)**

Given expansion is $\left(x + \frac{1}{2x}\right)^{2n}$

$\therefore$ Middle term $= {^{2n}C_n}(x)^n\left(\frac{1}{2x}\right)^n$

$= \frac{2n!}{n!\,n!\,2^n} = \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{n!}$

368 **(c)**

$(1+x)^{21} + (1+x)^{22} + \ldots + (1+x)^{30}$

$= (1+x)^{21}[1 + (1+x)^1 + \ldots + (1+x)^9]$

$= (1+x)^{21}\left[\frac{(1+x)^{10} - 1}{(1+x) - 1}\right]$

$= \frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$

$\therefore$ Coefficient of x^5 in the given expression

$=$ Coefficient of x^5 in $\frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$

$=$ Coefficient of x^6 in $[(1+x)^{31} - (1+x)^{21}]$

$= {^{31}C_6} - {^{21}C_6}$

369 **(a)**

We have,

$T_{r+1} = \frac{\frac{7}{2}\left(\frac{7}{2}-1\right)\left(\frac{7}{2}-2\right)\ldots\left(\frac{7}{2}-r+1\right)x^r}{r\,!}$

This will be the first negative term if

$\frac{7}{2} - r + 1 < 0 \Rightarrow r > \frac{9}{2}$

Hence, $r = 5$

370 **(b)**

According to question, coefficient of x^r =coefficient of x^{r+1}

$\Longrightarrow {^{21}C_r} = {^{21}C_{r+1}}$ ...(i)

But ${^{21}C_r} = {^{21}C_{21-r}}$ ...(ii)

On comparing Eqs. (i) and (ii), we get

$r + 1 = 21 - r \Longrightarrow r = \frac{21-1}{2} = 10$

371 **(a)**

Let $(r+1)^{th}$ term be the greatest term

We have,

$T_{r+1} = \sqrt{3} \cdot {^{20}C_r}\left(\frac{1}{\sqrt{3}}\right)^r$ and T_r

$= \sqrt{3}\ {^{20}C_{r-1}}\left(\frac{1}{\sqrt{3}}\right)^{r-1}$

$\therefore \frac{T_{r+1}}{T_r} = \frac{20-r+1}{r}\left(\frac{1}{\sqrt{3}}\right)$

$\therefore T_{r+1} \geq T_r$

$\Rightarrow 20 - r + 1 \geq \sqrt{3}\,r$

$\Rightarrow 21 \geq r(\sqrt{3}+1)$

$\Rightarrow r \leq \frac{21}{\sqrt{3}+1} \Rightarrow r \leq 7.686 \Rightarrow r = 7$

Hence, greatest term $T_8 = \sqrt{3}\,{^{20}C_7}\left(\frac{1}{\sqrt{3}}\right)^7 = \frac{25840}{9}$

372 **(b)**

We have,

$(1+x+x^2+\cdots)^{-n} = [(1-x)^{-1}]^{-n} = (1-x)^n$

$\therefore$ Coefficient of $x^n = (-1)^n\ {}^nC_n = (-1)^n$

373 **(a)**

We have, $(1+t^2)^{12}(1+t^{12})(1+t^{24})$

$= 1+{}^{12}C_1t^2 + {}^{12}C_2t^4 + {}^{12}C_3t^6 + \ldots + {}^{12}C_6t^{12} + \ldots)(1+t^{12}+t^{24}+t^{36})$

$\therefore$ Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24}) = {}^{12}C_6 + 2$

374 **(d)**

$T_r = {}^{14}C_{r-1}x^{r-1};\ T_{r+1} = {}^{14}C_rx^r;\ T_{r+2} = {}^{14}C_{r+1}x^{r+1}$

Since, these terms are in AP

$\therefore 2T_{r+1} = T_r + T_{r+2}$

$\Rightarrow 2\ {}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1}$ ...(i)

$\Rightarrow 2 \cdot \frac{14!}{r!\,(14-r)!} = \frac{14!}{(r-1)!\,(15-r)!} + \frac{14!}{(r+1)!\,(13-r)!}$

$\Rightarrow \frac{2}{r\cdot(r-1)!\,(14-r)\cdot(13-r)!} = \frac{1}{(r-1)!\cdot(15-r)\cdot(14-r)\cdot(13-r)!} + \frac{1}{(r+1)r(r-1)!\,(13-r)!}$

$\Rightarrow \frac{2}{r(14-r)} = \frac{1}{(15-r)(14-r)} + \frac{1}{(r+1)r}$

$\Rightarrow \frac{1}{r(14-r)} = \frac{1}{(15-r)(14-r)} + \frac{1}{(r+1)r} - \frac{1}{r(14-r)}$

$\Rightarrow \frac{(15-r)-r}{r(15-r)(14-r)} = \frac{(14-r)-(r+1)}{(r+1)r(14-r)}$

$\Rightarrow (15-r)-r = (13-2r)(15-r)$

$\Rightarrow 15r+15-2r^2-2r = 195-30r-13r+2r^2$

$\Rightarrow 4r^2-56r+180 = 0$

$\Rightarrow r^2-14r+45 = 0$

$\Rightarrow (r-5)(r-9) = 0 \Rightarrow r = 5,9$

But 5 is not given.

Hence, $r = 9$

375 **(c)**

We have, $(101)^{50} = (100+1)^{50}$

$= (100)^{50} + {}^{50}C_1(100)^{49} + {}^{50}C_2(100)^{48} + \cdots$ ....(i)

and $(99)^{50} = (100-1)^{50}$

$= (100)^{50} - {}^{50}C_1(100)^{49} + {}^{50}C_2(100)^{48} + \cdots$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$(101)^{50} - (99)^{50}$

$= 2\{{}^{50}C_1(100)^{49} + {}^{50}C_3(100)^{47} + \cdots\}$

$= 2\times {}^{50}C_1(100)^{49} + \{2\times {}^{50}C_3(100)^{47} + \cdots\}$

$= (100(100)^{49} + \text{a positive number})$

$> (100)^{50}$

$\therefore (101)^{50} > (100)^{50} + (99)^{50}$

376 **(a)**

The general term of the given series is

$T_r = (-1)^r(3+5r)\ {}^nC_r$

$\therefore$ Sum $\sum_{r=0}^{n}(1)^r\,(3+5r)\ {}^nC_r$

$= 3\sum_{r=0}^{n}(-1)^r\ {}^nC_r + 5\sum_{r=0}^{n}(1)^r\ r\,{}^nC_r$

$= 3(C_0 - C_1 + C_2 - C_3 + C_4 - \ldots + (-1)^n\cdot C_n)$

$+5(-C_1 + 2C_2 - 3C_3 + 4C_4 - \cdots + (-1)^n\cdot n\cdot C_n)$

$\Rightarrow S = 0+0 = 0$

378 **(d)**

$S = (\alpha\cdot 1 + \beta\cdot 1 + \gamma\cdot 1)^n = (\alpha+\beta-\gamma)^n$

$\therefore \lim_{n\to\infty}\frac{S}{\{S^{1/n}+1\}^n} = \lim_{n\to\infty}\left(\frac{\alpha+\beta-\gamma}{\alpha+\beta-\gamma+1}\right)^n = 0$

$(\because\ \alpha+\beta-\gamma+1 > \alpha+\beta-\gamma)$

379 **(b)**

It is given that the sum of the numerical coefficients in the binomial expansion of $\left(\frac{1}{x}+2x\right)^n$ is 6561

$\therefore (1+2)^n = 6561$ [Putting $x = 1$]

$\Rightarrow 3^n = 3^8 \Rightarrow n = 8$

The general term in the expansion of $\left(\frac{1}{x}+2x\right)^n$ is given by

$T_{r+1} = {}^nC_r\left(\frac{1}{x}\right)^{n-r}(2x)^r = {}^nC_r2^rx^{-n+2r}$

$= {}^8C_r2^r\,x^{2r-8}$

This will be independent of x id $r = 4$

Hence, the constant term $= {}^8C_42^4$

381 **(c)**

Multiplying the numerator and denominator by $1-x$, we have $E = \frac{1-x}{(1-x)(1+x)(1+x^2)(1+x^4)\ldots(1+x^{2^m})}$

$= \frac{1-x}{(1-x^2)(1+x^2)(1+x^4)\ldots(1-x^{2^m})}$

$= \frac{1-x}{(1-x^4)(1+x^4)\ldots(1-x^{2^m})}$

$= \frac{1-x}{(1-x^{2^{m+1}})} = (1-x)(1-x^{2^{m+1}})^{-1}$

$= (1-x)(1+x^{2^{m+1}}+x^{2^{m+2}}+\dots)$

$\therefore$ Coefficient of $x^{2^{m+1}}$ is 1

382 **(b)**

Let $G=(7-4\sqrt{3})^n$. Then,

$0<G<1$ as $0<7-4\sqrt{3}<1$

Now,

$I+F+G=(7+4\sqrt{3})^n+(7-4\sqrt{3})^n$

$\Rightarrow I+F+G=2({}^nC_0\,7^n+{}^nC_2\,7^{n-2}(4\sqrt{3})^2+\cdots)$

$\Rightarrow I+F+G=$ an integer

$\Rightarrow F+G=1$

$\Rightarrow G=1-F$

$\therefore (I+F)(1-F)=(I+F)G$

$=(7+4\sqrt{3})^n(7-4\sqrt{3})^n=1$

383 **(c)**

Since, number of terms in the expansion of $(1+x)^{24}$ is 25.

Therefore, the middle term is 13th term.

$\therefore$ Required greatest coefficient $={}^{24}C_{12}$.

384 **(d)**

We have,

$(1+x+x^3+x^4)^{10}$

$=(1+x)^{10}(1+x^3)^{10}$

$=({}^{10}C_0+{}^{10}C_1\,x+{}^{10}C_2\,x^2+{}^{10}C_3\,x^3+{}^{10}C_4\,x^4+\cdots+{}^{10}C_{10}\,x^{10})\times({}^{10}C_0+{}^{10}C_1\,x^3+{}^{10}C_2\,x^6+\cdots+{}^{10}C_{10}\,x^{30})$

$\therefore$ Coefficient of $x^4={}^{10}C_0\times{}^{10}C_4+{}^{10}C_1\times{}^{10}C_1$

$=310$

385 **(b)**

We have,

$(1+x)^m=1+m\,x+\frac{m(m-1)}{2\,!}x^2+\cdots$

It is given that the third term is $-\frac{1}{8}x^2$

$\therefore \frac{m(m-1)}{2}x^2=-\frac{1}{8}x^2$

$\Rightarrow 4\,m^2-4\,m=-1\Rightarrow(2\,m-1)^2=0\Rightarrow m=\frac{1}{2}$

386 **(a)**

We have,

$(C_0+C_1)(C_1+C_2)(C_2+C_3)\dots(C_{n-1}+C_n)$

$=C_1\,C_2\dots C_{n-1}C_n\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_1}{C_2}\right)\left(1+\frac{C_2}{C_3}\right)\dots\left(1+\frac{C_{n-1}}{C_n}\right)$

$=C_1\,C_2\dots C_{n-1}\,C_n\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{2}{n-1}\right)\left(1+\frac{3}{n-2}\right)\dots\left(1+\frac{n}{1}\right)$

$=C_1\,C_2\dots C_{n-1}\,C_n\frac{(n+1)^n}{n\,!}$

$\therefore k=C_1\,C_2\,C_3\dots C_{n-1}\,C_n=C_0C_1C_2\dots C_{n-1}\,C_n$

387 **(c)**

We have,

$\sum_{m=0}^{100}{}^{100}C_m\,(x-3)^{100-2}2^m=[(x-3)+2]^{100}$

$=(1-x)^{100}$

$\therefore$ Coefficient of $x^{53}={}^{100}C_{53}(-1)^{53}=-{}^{100}C_{53}$

388 **(d)**

Given expansion is $\left(x-\frac{1}{2x}\right)^n$

$\therefore T_3={}^nC_2(x)^{n-2}\left(-\frac{1}{2x}\right)^2$

and $T_4={}^nC_3(x)^{n-3}\left(-\frac{1}{2x}\right)^3$

But according to the given condition,

$\frac{T_3}{T_4}=-\frac{n(n-1)\times3\times2\times1\times8}{n(n-1)(n-2)\times2\times1\times4\times}=\frac{1}{2}$ (given)

$\Rightarrow\ -n+2=12\Rightarrow n=-10$

389 **(b)**

We have, $(1+x+x^2+\dots)^{-n}=\left(\frac{1}{1-x}\right)^{-n}=(1-x)^n$

$\therefore$ The coefficient of x^n is $(-1)^n$

390 **(b)**

Here, $T_4={}^nC_3(a)^{n-3}(-2b)^3$

and $T_5={}^nC_4(a)^{n-4}(-2b)^4$

Given, $T_4+T_5=0$

$\Rightarrow\ {}^nC_3(a)^{n-3}(-2b)^3+{}^nC_4(a)^{n-4}(-2b)^4=0$

$\Rightarrow\ (a)^{n-4}(-2b)^3[a\,{}^nC_3+{}^nC_4(-2b)]=0$

$\Rightarrow\ \frac{a}{b}=\frac{2\,{}^nC_4}{{}^nC_3}$

$=\frac{2.n(n-1)(n-2)(n-3)}{4.3.2.1}\times\frac{3.2.1}{n(n-1)(n-2)}$

$=\frac{n-3}{2}$

391 **(d)**

The general term in the expansion of $(2-x+3x^2)^6$ is given by

$\frac{6!}{r!\,s!\,t!}2^r(-x)^s(3x^2)^t$, where $r+s+t=6$

$=\frac{6!}{r!\,s!\,t!}2^r\times(-1)^s\times3^t\times x^{s+2t}$, where $r+s+t=6$

For the coefficient of x^5, we must have $s+2t=5$

But, $r+s+t=6$

$\therefore s=5-2t$ and $r=1+t$, where $0\le r,s,t\le6$

Now,

$t=0\Rightarrow r=1, s=5$

$t = 1 \Rightarrow r = 2, s = 3$
$t = 2 \Rightarrow r = 3, s = 1$
Thus, there are three terms containing x^5 and hence
Coefficient of x^5

$= \frac{6!}{1!\,5!\,0!} \times 2^1 \times (-1)^5 \times 3$
$+ \frac{6!}{2!\,3!\,1!} \times 2^2 \times (-1)^3 \times 3^1 + \frac{6!}{3!\,1!\,2!} \times 2^3 \times (-1)^1 \times 3^2$
$= -12 - 720 - 4320 = -5052$

392 **(b)**
We have,
$\left(x^2 - 2 + \frac{1}{x^2}\right)^n = \left(x - \frac{1}{x}\right)^{2n}$
$\therefore T_{r+1} = {}^{2n}C_r\, x^{2n-r} \left(-\frac{1}{x}\right)^r = {}^{2n}C_r\, x^{2n-2r}(-1)^r$
This term will be independent of x if $2n - 2r = 0$ i.e. $r = n$
$\therefore$ Number of terms dependent on $x = (2n + 1) - 1 = 2n$

393 **(b)**
We have,
Coeff. of x^7 = Coeff. of x^8
$\Rightarrow {}^nC_7 \times 2^{n-7} \times \left(\frac{1}{3}\right)^7 = {}^nC_8 \times 2^{n-8} \times \left(\frac{1}{3}\right)^8$
$\Rightarrow 6({}^nC_7) = {}^nC_8 \Rightarrow 48 = n - 7 \Rightarrow n = 55$

394 **(c)**
We have,
$\therefore T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2\,x^2}\right)^r$
$\Rightarrow T_{r+1} = {}^{10}C_r \left(\frac{1}{3}\right)^{5-\frac{r}{2}} \left(\frac{3}{2}\right)^r x^{5-\frac{5r}{2}}$
For this term to be independent of x, we must have
$5 - \frac{5r}{2} = 0 \Rightarrow r = 2$, which is an integer
Hence, third term is independent of x
Also, $T_3 = {}^{10}C_2 \left(\frac{1}{3}\right)^4 \left(\frac{3}{2}\right)^2 = 45 \times \frac{1}{81} \times \frac{9}{4} = \frac{5}{4}$

395 **(d)**
Suppose x^{-4} occurs in $(r + 1)^{\text{th}}$ term
We have,
$T_{r+1} = {}^{10}C_r \left(\frac{3}{2}\right)^{10-r} \left(\frac{-3}{x^2}\right)^r$
$= {}^{10}C_r \left(\frac{3}{2}\right)^{10-r} (-3)^r\, x^{-2r}$
This will contain x^4, if $-2\,r = -4 \Rightarrow r = 2$
$\therefore$ Coefficient of $x^{-4} = {}^{10}C_2 \left(\frac{3}{2}\right)^{10-2} (-3)^2$
$= \frac{3^{12} \times 5}{2^8}$

396 **(b)**
By hypothesis, we have
${}^{18}C_{2\,r+3} = {}^{18}C_{r-3} \Rightarrow 2\,r + 3 = r - 3 \Rightarrow r = 6$

398 **(d)**
$49^n + 16n - 1 = (1 + 48)^n + 16nn - 1$
$= 1 + n_{C_1}(48) + n_{C_2}(48)^2 + \ldots + n_{C_n}(48)^n + 16n - 1$
$= (48n + 16n) + n_{C_2}(48)^2 + n_{C_3}(48)^3 + \ldots + n_{C_n}(48)^n$
$64n + 8^2(n_{C_2} \cdot 6^2 + n_{C_3} \cdot 6^3 \cdot 8 + n_{C_4} \cdot 6^4 \cdot 8^2 + \ldots + n_{C_n} \cdot 6^n \cdot 8^{n-2})$
Hence, $49^n + 16n - 1$ is divisible by 64
Alternate Let $P(n) = 49^n + 16n - 1$
For $n = 1$
$P(1) = 49 + 16 - 1 = 64$

399 **(d)**
We have
$\sum_{r=1}^{n} r^2 \cdot {}^nC_r = n(n-1)2^{n-2} + n \cdot 2^{n-1}$ ... (i)
and $\sum_{r=1}^{n} (-1)^{r-1} r^2\, {}^nC_r = 0$ ...(ii)
On adding Eqs. (i) and (ii), we get
$2[1\,{}^2C_1 + 3^2C_3 + 5^2C_5 + \cdots]$
$= n(n-1)2^{n-2} + n \cdot 2^{n-1}$
$\Rightarrow 1^2C_1 + 3^2C_3 + 5^2C_5 + \ldots$
$= n(n-1)2^{n-3} + n \cdot 2^{n-2}$

400 **(d)**
Since, $(3 + ax)^9 = {}^9C_0 3^9 + {}^9C_1 3^8(ax) + {}^9C_2 3^7(ax)^2 + {}^9C_3 3^6(ax)^3 + \cdots$
Since, coefficient of x^2 = coefficient of x^3
$\Rightarrow \quad {}^9C_2 3^7 a^2 = {}^9C_3 3^6 a^3$
$\Rightarrow \quad \frac{{}^9C_2}{{}^9C_3} \cdot 3 = a$
$\Rightarrow \quad \frac{\frac{9\times8}{2\times1}}{\frac{9\times8\times7}{3\times2}} \times 3 = a$
$\Rightarrow \quad a = \frac{9}{7}$

401 **(a)**
The coefficient of x in the expansion of $(1 + x)(1 + 2x)(1 + 3x)\ldots(1 + 100x)$
$= 1 + 2 + 3 + \ldots + 100$
$\frac{100(100 + 1)}{2} = 50 \times 101 = 5050$

402 **(a)**

Suppose x^5 occurs in $(r+1)^{\text{th}}$ term of the expansion of $\left(x^2+\frac{a}{x^3}\right)^{10}$

We have,

$T_{r+1} = {}^{10}C_r\,(x^2)^{10-r}\left(\frac{a}{x^3}\right)^r = {}^{10}C_r\,x^{20-5r}\,a^r$

$\therefore 20-5r=5 \Rightarrow r=3$

$\therefore$ Coefficient of $x^5 = {}^{10}C_3\,a^3$

Similarly, Coefficient of $x^{15} = {}^{10}C_1\,a^1$

Now,

Coeff. of x^5 = Coeff. of x^{15}

$\Rightarrow {}^{10}C_3\,a^3 = {}^{10}C_1\,a$

$\Rightarrow 120\,a^3 = 10\,a \Rightarrow a^2 = \frac{1}{12} \Rightarrow a = \frac{1}{2\sqrt{3}}$

403 **(a)**

$\because$ 10th term in the expansion of $(2-3x^3)^{20}$ is ${}^{20}C_9 2^9(-1)^9(2)^{11}(3)^9x^{27}$ and 11th term is ${}^{20}C_{10}2^{10}3^{10}x^{30}$

$$\therefore \quad \frac{{}^{20}C_9(-1)^9(2)^{11}(3)^9x^{27}}{{}^{20}C_{10}\cdot 2^{10}\cdot 3^{10}\cdot x^{30}} = \frac{45}{22}$$

$$\Rightarrow \quad -\frac{10}{11}\cdot\frac{2}{3x^3} = \frac{45}{22}$$

$$\Rightarrow \quad x^3 = -\frac{8}{27} \Rightarrow x = -\frac{2}{3}$$